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# Simulation of Un-damped and Damped Oscillations in RLC Circuit using MATLAB Computer program

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**ABSTRACT**
In this paper, we elaborate the characteristics of oscillations in RLC circuit using MATLAB computer program. To study the characteristics we apply second order differential equations to obtain the expression of electric charge resulted from Kirchhoff's loop rule and use the solution of it to determine the expressions for electric current (i), energy stored in capacitor and energy stored in the inductor. In the simplest case, the resonant circuit consists only of a capacitor C and inductor L, and characterizes un-damped electrical oscillations. An RLC circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor(C), connected in series or in parallel, and characterizes damped oscillations. For both cases, the un-damped and damped oscillations, we compare the graphs of circuit without resistance with the graphs of circuit with resistance obtained using MATLAB computer program.

**Keywords:** *RLC circuit, LC circuit, Kirchhoff's loop rule, un-damped and damped oscillations*

## 1. INTRODUCTION

It is worth noting that both capacitors and inductors store energy, in their electric and magnetic fields, respectively. A circuit containing both an inductor (*L*) and a capacitor (*C*) can oscillate without a source of emf by shifting the energy stored in the circuit between the electric and magnetic fields. Thus, the concepts we develop in this section are directly applicable to the exchange of energy between the electric and magnetic fields in electromagnetic waves. [2].If the capacitor is initially charged and the switch is then closed, both the current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values. If the resistance of the circuit is zero, no energy is transformed to internal energy. If a resistor is connected in series with the capacitor and inductor we will provide a method of energy loss in the system .Therefore, the oscillations of charge, current, and potential difference continuously decrease in amplitude and eventually damp out to zero.

### 1.1 Un-damped Oscillations

In the simplest case, the resonant circuit consists only of a capacitor C and inductor L, and characterizes un-damped electrical oscillations. When a capacitor is connected to an inductor as illustrated in Figure 1, the combination is an LC circuit. If the capacitor is initially charged and the switch is then closed, both the current in the circuit and the charge on the capacitor oscillate between maximum positive and negative values. If the resistance of the circuit is zero, no energy is transformed to internal energy. The presence of inductance in an electric circuit gives the current an "inertia" (resistance to change which causes the damping.), since inductors try to prevent changes in the flow of current. The presence of capacitance in a circuit means that charge can flow into one side of the capacitor to be stored there, and later this charge can restore the electric current as the capacitor discharges. Thus, an *LC* circuit can oscillate without a source of emf by shifting the energy stored in the circuit between the electric and magnetic fields[1-4].

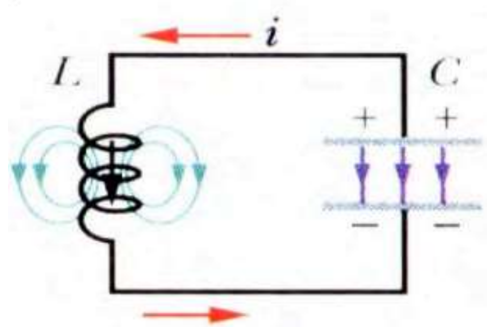


Figure 1:A simple LC circuit[7]

When the capacitor is fully charged, the energy U in the circuit is stored in the electric field of the capacitor. At this time, the current in the circuit is zero, and thus no energy is stored in the inductor. After the switch is thrown closed, the rate at which charges leave or enter the capacitor plates (which is also the rate at which the charge on the capacitor changes) is equal to the current in the circuit. As the capacitor begins to discharge after the switch is closed, the energy stored in its electric field decreases. The discharge of the capacitor represents a current in the circuit, and hence some energy is now stored in the magnetic field of the inductor. Thus, energy is transferred from the electric field of the capacitor to the magnetic field of the inductor. When the capacitor is fully discharged, it stores no energy. At this time, the current reaches its maximum value, and all of the energy is stored in the inductor. The current continues in the same direction, decreasing in magnitude, with the capacitor eventually becoming fully charged again but with the polarity of its plates now opposite the initial polarity. The energy continues to oscillate between inductor and capacitor[1,3,4,8].

To study this oscillation in detail, we apply Kirchhoff's loop rule to the circuit in Fig.1.which states that "the sum of the voltages around any loop of the circuit is zero" [1,3,5,6]

$v_L + v_C = 0$------------------------------------ (1)

$-L\frac{dI}{dt} - \frac{Q}{C} = 0$ ---------------------------------- (2)

As we know the current in the circuit is equal to the rate at which the charge on the capacitor changes: $I = dQ/dt$ .Then we can reduced to one variable. From this, it follows that

$\frac{dI}{dt} = \frac{d^2Q}{dt^2}$…………………..……………… (3)

Substituting equ.(3) in to equ.(2), we get

$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$ ……………………………… (4)

Equation (4) is linear, second order and homogeneous ordinary differential equation (ODE).

If we denote the $\frac{1}{\sqrt{LC}}$ with the symbol$\omega$, $\omega^2 = \frac{1}{LC}$

$\omega = \frac{1}{\sqrt{LC}}$ -------------------------------------- (5)

Accordingly, $\frac{d^2Q}{dt^2} = -\omega^2 Q$------------- (6)

The Mathematical solution to the above differential equation is

$Q(t) = Q_{max}cos(\omega t + \phi)$--------------------- (7)

Where$Q_{max}$, $\omega$, and $\phi$are constants.

The period $T$ of the oscillation in LC circuit is

$T = 2\pi\sqrt{LC}$--------------------------------- (8)

The inverse of the period is called the frequency $f$ of the oscillation.

$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}$---------------------------------- (9)

For the given harmonically oscillating charge, the voltage and the current in the LC circuit also oscillate according to equations.

$$v = \frac{Q}{C} = \frac{Q_{max}}{C}\cos(\omega t + \phi)$$

$V = V_{max}\cos(\omega t + \phi)$------------------------ (10)

An electric circuit driven by a periodic external voltage whose frequency matches the natural frequency of oscillations, then the system is said to be “in resonance” with the driving voltage and the amplitude of oscillations can grow very large.

$$I = \frac{dQ(t)}{dt} = Q_{max}\frac{d}{dt}[cos(\omega t + \phi)]$$

$I = -\omega Q_{max}sin(\omega t + \phi)$--------------------- (11)

Where$\omega Q_{max} is$ maximum current and is given by

$I_{max} = \omega Q_{max} = Q_{max}\frac{1}{\sqrt{LC}}$------------------ (12)

With these solutions, the electric potential energy stored in the capacitor is given by

$U_C = \frac{Q^2}{2C}cos^2(\omega t + \phi)$------------------------ (13)

And the magnetic energy stored in the inductor,

$U_L = \frac{LI^2}{2}sin^2(\omega t + \phi)$------------------------ (14)

Combining equations (13) and (14) the total electrical energy of the LC oscillator can be expressed as

$U = U_C + U_L = \frac{1}{2C}{Q_{max}}^2[sin^2(\omega t + \phi) + cos^2(\omega t + \phi)]$------- ---------------------- (15)

Therefore, the equation reduces to

$U = \frac{1}{2C}{Q_{max}}^2$---------------------------------- (16)

This expression contains all of the features described qualitatively at the beginning of this section. It shows that the energy of the LC circuit continuously oscillates between energy stored in the electric field of the capacitor and energy stored in the magnetic field of the inductor.

**1.2 Damped Oscillation**

Suppose an inductor with inductance L and a resistor of resistance R are connected in series across the terminals of a charged capacitor, forming an L-R-C series circuit. If we add a resistor in series with the capacitor and inductor we will provide a method of energy loss in the system. Whenever current flows some energy will be lost to heat in the resistor and the oscillations will eventually damp out to zero. Thus, the oscillations of charge, current, and potential difference continuously decrease in amplitude, and the oscillations are said to be damped [1,4].

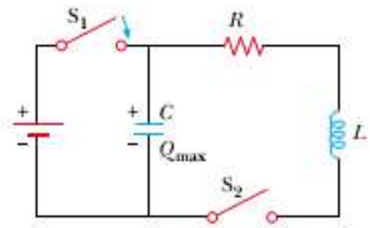


Figure 2: A series RLC circuit[1]

To analyze L-R-C series circuit behavior in detail, consider the circuit shown in Fig. 2. It is like the Circuit of Fig. 1 except for the added resistor R; we also show the source that charges the capacitor initially. The labeling of the positive senses of Q and I is the same as for the LC circuit.

First we close the switch in the upward position, connecting the capacitor to a source of emf ε for a long enough time to ensure that the capacitor acquires its final charge Q=Cε and any initial oscillations have died out. Then at time t =0 we flip the switch to the downward position, removing the source from the circuit and placing the capacitor in series with the resistor and inductor.

To find how Q and I vary with time, we apply Kirchhoff’s loop rule and obtain

$-IR - L\frac{dI}{dt} - \frac{Q}{C}$=0------------------------------ (17)

Replacing $I$with $I = \frac{dQ}{dt}$ and rearranging, we get

$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0$-------------------------- (18)

There are general methods for obtaining solutions of Eq. (18).

When $R^2$is less than $4L/C$, the solution has the form

$Q(t) = Q_{max}e^{-\left(\frac{R}{2L}\right)t}cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}t + \phi\right)$-- (19)

The angular frequency $\omega'$ of the damped oscillations is given by

$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

Where$Q_{max}$and $\phi$are constants.

The form of the solution is different for the under-damped (small R), over-damped (large R) and critical damped. This solution corresponds to the function represents a sinusoidal oscillation with exponentially decaying amplitude [2]

(1) For$\frac{R^2}{4L^2} > \frac{1}{LC}$ , ω is an imaginary number and cos (ωt) = cosh (jωt), which is a hyperbolic cosine function. There is no oscillation. It's called “over damped".

(2) For$\frac{R^2}{4L^2} = \frac{1}{LC}$, ω= 0 and cos (ωt) = 1. There is no oscillation, either. It's called "critical damped".

(3) For$\frac{R^2}{4L^2} < \frac{1}{LC}$, ω is a real number. Oscillation occurs. It's called “under damped". If $\frac{R^2}{4L^2} \ll \frac{1}{LC}$, the frequency of the

oscillation would approach $\omega=\frac{1}{\sqrt{LC}}$, called nature angular frequency.

**Result and discussion**

In the following analysis, we use the solutions of un-damped and damped oscillations and compare the graphs of circuit without resistance with the graphs of circuit with resistance obtained using MATLAB computer program.

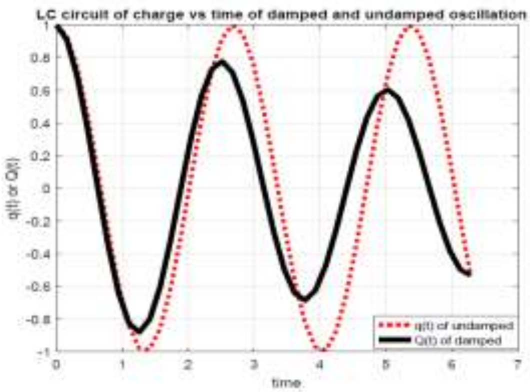


Figure 3: Simulation of charge for un-damped and damped circuit oscillation using MATLAB

Here, in figure 3 the red broken line represents the graph of charge versus time of ideal case. It shows that the exact behavior of un-damped oscillation, that is, the amplitude doesn't depend on time it remains constant with time. And the black line represents the graph of charge versus time of real case. It shows that the exact behavior of damped oscillation, that is, the amplitude depends on time so that we see that the amplitude may decay in time due the overall factor of $e^{-\left(\frac{R}{2L}\right)t}$.

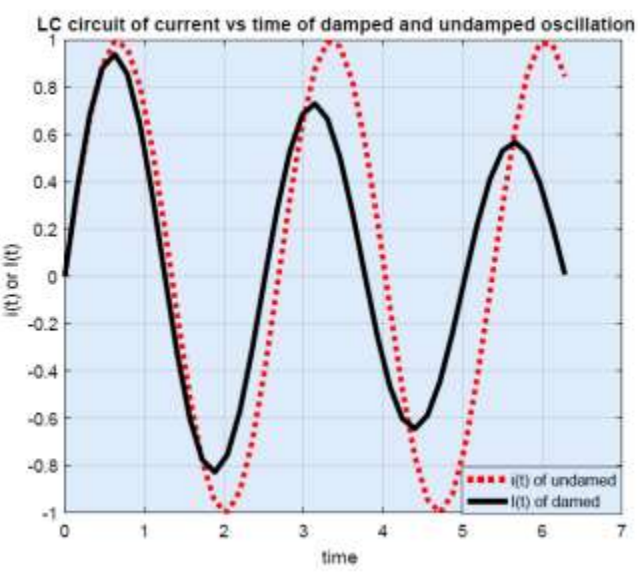


Figure 4: Simulation of current for un-damped and damped circuit oscillation using MATLAB

As used in figure 3, the red broken line represents the graph of current versus time of un-damped oscillation and the black line represents the graph of current versus time of damped oscillation. The ideal circuit has no resistance, the peak current remains constant. Real circuit has resistance, Peak current decays with time constant = 2L/R

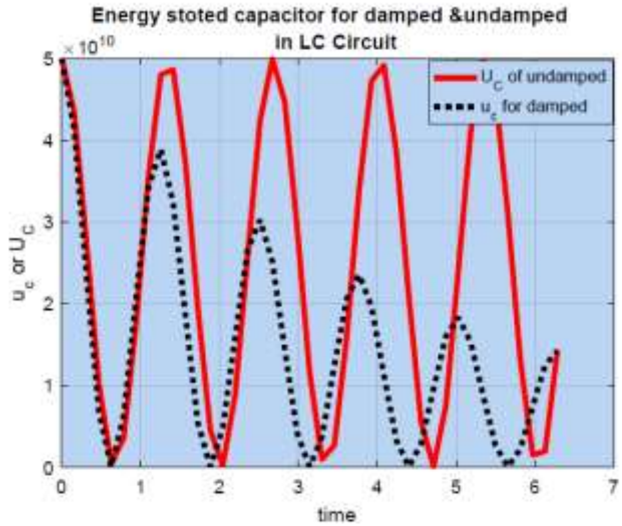


Figure 5: Simulation of energy stored in the capacitor for un-damped and damped circuit oscillation using MATLAB

Here, the red line represents the graph of energy stored in the capacitor versus time of un-damped oscillation and the black broken line represents the graph energy stored in the capacitor versus time of damped oscillation. As indicated in figure, the amplitude energy stored in the inductor remains constant for un-damped and decreases exponentially with time duet to the effect of resistance in the circuit.

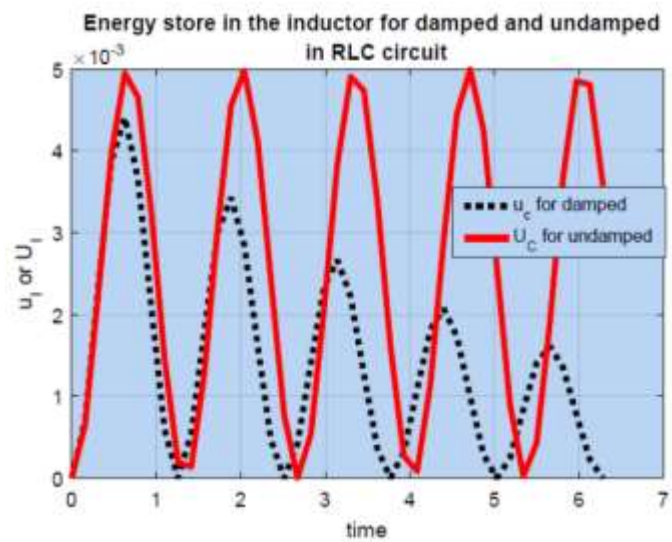


Figure 6: Simulation of energy stored in the inductor for un-damped and damped circuit oscillation using MATLAB

Here also, the red line represents the graph of energy stored in the capacitor versus time of un-damped oscillation and the black broken line represents the graph energy stored in the inductor versus time of damped oscillation. As indicated in figure, the amplitude energy stored in the inductor remains constant for un-damped and the amplitude of the oscillation appears to decrease with time and becomes zero after some time for damped one.

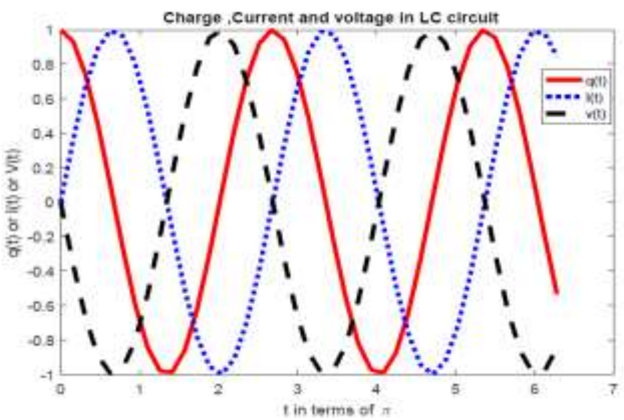


Figure 7: Plots of charge, current and voltage versus t for a resistance less, non-radiating LC circuit.

In this case the amplitudes of charge, current and voltage remain constant. This holds true for all LC circuits (résistance less circuit). Therefore, the amplitudes of the three graphs in Figure 7 must be equal.

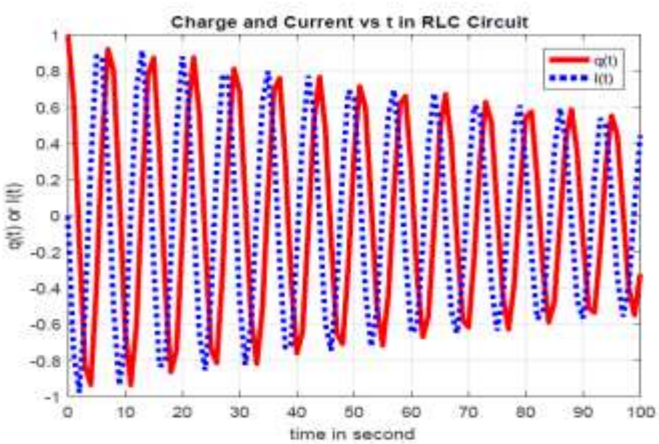


Figure 8: Plots of charge and current versus t for RLC circuit

As the charge and current contained in the circuit oscillate back and forth through the resistance, the energies stored in capacitor and inductor are dissipated and cause damping (decreasing the amplitude of) the oscillations. So that the total energy stored in the circuit continuously decreases as a result of this process.

## CONCLUSION

In LC circuit (one with no resistor) charge flows back and forth between the capacitor plates through the inductance. The energy oscillates back and forth between the capacitor's electric field(E) and the inductor's magnetic field (B). The resonant frequency for such a circuit is un-damped resonance frequency. In RLC circuit (one with resistor), the resistor increases the decay of the oscillation and reduces the peak resonant frequency. Therefore, the peak resonance frequency depends on the value of the resistor and is described as the damped resonant frequency. The value of resistor causes the circuit oscillations to be critically damped, under damped and over damped. Despite of this, RLC circuits operate the same way as LC circuits, except that the oscillating currents decay with time to zero due to the resistance in the circuit.

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