Research Article **Open Access**

Design of Analog PID Controller

[1]**Theophilus, E.E.,** [2]**Ismail, I.,** [2]**Fagbohun, O.O.,** [2]**Femi-Jemilohun, O.J.**

[1] Department of Electrical and Electronic Engineering, Ogun State Institute of Technology, Igbesa, Nigeria

[2] Department of Electrical and Electronic Engineering, Ekiti State University, Ado-Ekiti, Nigeria.

*Corresponding author: Theophilus, E.E, E-mail: ddarasimi9@gmail.com

ABSTRACT

The PID controller has been used and dominated the process control industries for a long time as it provides the control action in terms of compensation based on present error input(proportional control), on past error(integral control) and on future error if recorded by earlier experience or some means(derivative control). The PID controllers have excellent property of making the system response faster and at the same time reduce the steady state error to zero or at least to a very small tolerance limit. The work below starts with study of individual components of the controller and their responses in a certain environment for different test signals (say a step or sine wave input).The problem is to design a PID controller using appropriate analog circuit as well as understand and utilize the advantages of all the three terms. The below work is for the study of an analog PID controller using operational amplifiers and fabricate the controller on hardware after testing the individual terms:-proportional, integral and derivative.

Keywords: *Bio Fuels, HCCI, stratified charge compression ignition (SCCI), low temperature combustion (LTC), Emissions, Simulation*

INTRODUCTION

The process control literature is replete with theory and application for linear systems. However, it is a well-known fact that virtually all practical industrial process systems exhibit nonlinear dynamic behavior and for these, the linear techniques are not directly applicable. A proportional-integral-derivative (PID) controller is one of the commonly used controllers in process industries (almost all process control today contain more than 95 % PID controllers) and are for controlling feedback systems because of the reduced number of parameters to be tuned. These controllers provide control signals that are proportional to the error between the reference signal and the proportional action (actual output), to the integral action (integral of the error), and to the derivative action (derivative of the error)[1,2,3,8]

The electronic controllers are replacing the conventional pneumatic controllers after the development of electronic devices and operational amplifiers. Nowadays, the main focus of the development is the implementation with digital PID controllers due to the advent of the microprocessors and microcontrollers [4]. The highest advantage of using a digital PID controllers is that the controllers' parameters can be programmed smoothly; thus, without changing any hardware, the controllers can be changed. Additionally, other than generating the control action, the same digital system can be used for a number of other applications [4,5,6,7,8]

This study is concerned with the design of analog PID controller, and how it can be implemented in real practice. Designing automatic control systems is perhaps the most important task that a control engineer carries out. We may investigate and find out approaches to do the design in certain cases while mostly we do design based on trial and error basis.

This requires that we should put some restrictions and constraints along with pre-determined performance conditions in order to get a better quality control in terms of performance. So, controller design requires various factors to be taken care of.

A control system designed for a specification or specific application has to meet certain performance specifications. Some approaches determining the performance of a control system are by set of specifications in time domain and/or in frequency domain such as peak overshoot, settling time, gain margin, phase margin, steady-state error etc. and also, by optimality of a certain function, e.g., an integral function.

In this study, the choice of plant segments to be controlled is dictated by performance as well as size, weight, accessible power supply, cost etc. In this way, the plant generally cannot meet the performance details. However, the designer is allowed to pick alternative components, this is generally not done on account of cost, accessibility and different requirements.

Though, a few components of a plant, its substitution are not a major issue in view of ease and extensive variety of availability of such amplifiers. Just by gain modification, it might be conceivable to meet the given details on performances of simple control systems. In such cases, gain adjustments appears to be the most straight and simple method for design. In most cases, the gain tuning does not give the desired result. Under such conditions, it is important to present some sort of appropriate subsystems to drive the chosen plant to meet the precise performances. These subsystems are known as controllers/compensators and their job is to make up for the lack in the performances of the plant.

DESIGN METHOD

The control system design problem have two fundamental methods (1). the structure of the overall system is selected by introducing controller and then the performance parameters of the controller is chosen to meet the given conditions on performance and (2). in a given plant, the overall system is inspected in order to meet the given specification and hence, compute the required controller.

The first methodology will be used in this work. Thus, the plant components are determined considering various factors; the plant cannot meet these conditions. For this reason, gain change seems suitable, as replacement by other components may be costly or

unrealistic. This is because the steady state error transfer function is inversely proportional to open loop gain and is given by:

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} \tag{1}$$

Where

G(s) = open loop transfer function or gain

However gain tuning using such Proportional gain (P) leads to oscillatory transient response and may lead to instability, although it decreases steady state error to a desirable tolerance limit.

Therefore, a PID controller can be utilized which has the advantage of making the system response faster, reduce the steady state error to zero or within a bearable limit. The PID controller is avoided in some process industries in recent time and PI controller is preferred because the derivate control poses some problems. In this study, each of the control parameters; proportional, integral, derivative, individually or with combination as PD or PI are to be designed to give a PID controller on hardware for an arbitrary plant using suitable tuning methods and meanwhile understand the advantages that can be more prominent and utilized for a particular specification. The major difficulty with PID control is that it is a feedback system, with constant parameters, and do have a direct knowledge of the process, consequently giving overall performance is reactive and a compromise.

ANALOG PID IMPLEMENTATION

The implementation of the PID controller can use both digital and/or analog circuits. Digital PID is implemented using integrated circuits while various circuits using operational amplifiers is the case of analog design as shown in Figure 2.

From Figure 3, it is seen that there are basically three different inverter circuits with different values of impedances $Z_f(s)$ and $Z_i(s)$. The inverter circuit is shown in Figure 1.
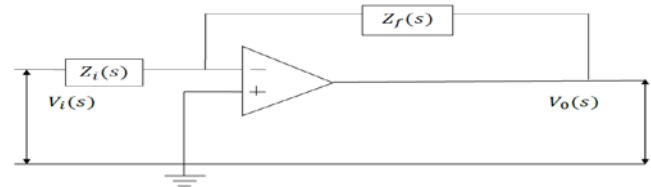

Figure 1: An inverter circuit

The above inverter circuit has a closed loop gain given by:

$$G(s) = -\frac{Z_f(s)}{Z_i(s)} \tag{2}$$

For different values of $Z_f(s)$ and $Z_i(s)$, we can get various control actions and thus implement different types of controllers as shown in Table 1.

Table 1: Transfer function of various controllers using Op Amps

Controller	Z_f	Z_i	Transfer Function G(s)
P	R_f	R_i	$-\dfrac{R_f}{R_i}$
PI	$R_f + \dfrac{1}{sC_f}$	R_i	$-\left[\dfrac{R_f}{R_i} + \dfrac{1}{sC_f R_i}\right]$
PD	R_f	$\dfrac{R_i}{sC_i R_i +1}$	$-\left[\dfrac{R_f}{R_i}(sC_i R_i +1)\right]$
PID	$R_f + \dfrac{1}{sC_f}$	$\dfrac{R_i}{(sC_i R_i +1)}$	$-\left[\dfrac{(sC_i R_i +1)(sC_f R_f +1)}{(sC_f R_i)}\right]$

Hence, the transfer functions using Op Amp for PID controller can be as in Table 1 is:

$$-\left[\frac{(sC_i R_i +1)(sC_f R_f +1)}{(sC_f R_i)}\right] \tag{3}$$

Or

The transfer function can take following shape as per the diagram shown in Figure 3 as follows:

$$G(s) = -\left[\frac{R_{p2}}{R_{p1}} + \frac{sC_D R_D}{sC_D R_C +1} + \frac{1}{sC_I R_I}\right] \tag{4}$$

This circuit contains a summer circuit that sums up command signal generated by each of the control terms and finally an inverter is used for getting positive value of transfer function.
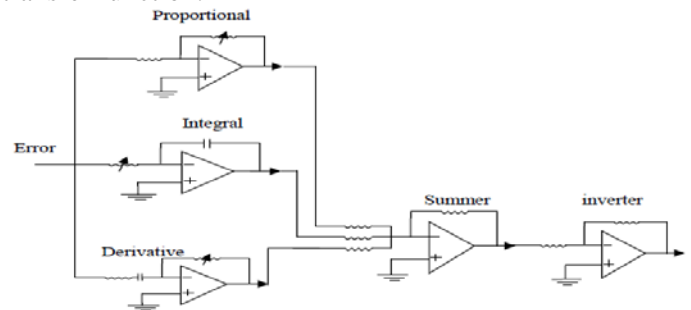

Figure 2: Circuit diagram of a PID controller

PROPERTIES OF GAIN PARAMETERS ON PERFORMANCE

Considering a second order system and its overall transfer function for a closed loop second order system is of this form:

$$\frac{C(s)}{R(s)} = \frac{w_n^2}{s^2 + 2 + w_n^2} \tag{5}$$

The study of second order systems is important because it is simpler and higher order systems can be approximated to a fair extent by second order systems and thus, one can get fair idea about the dynamics of the system and steady state error. The dynamics refers to the response of a system response to an abnormal condition such as lightning, sudden rise of voltage, constantly increasing input etc., and such systems are studied using test signals like impulse, step, ramp etc.

The dynamics can be analyzed by knowing the damping $(\Im)$ and undamped natural frequency (w_n). This can give known from the system response viz., peak overshoot (M_p), rise time (t_r), settling time (t_s), steady-state error (e_{ss}).

For a step input, these values are given by following equations:-

1. Rise time (t_r) is the time required by response to rise from 10 % to 90 % of final value for overdamped system and 0 to 100 % for underdamped system.

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\Im^2}}{\Im}}{w_n\sqrt{1-\Im^2}} \tag{6}$$

2. Peak overshoot (M_p), is normalized difference between peak of response and steady state output normalized w.r.t. to steady output.

$$M_p = e^{-\frac{\pi\Im}{\sqrt{1-\Im^2}}} \tag{7}$$

3. Settling time (t_s) is the time required for the response to reach and stay within specified limit of its final value called tolerance band (2-5 %). This value is for 5 % band,

$$t_s = \frac{4}{\Im w_n} \tag{8}$$

4. Steady state error (e_{ss}) is the error between the actual output and desired output as t tends to infinity.

$$e_{ss} = \frac{2\Im}{w_n} \tag{9}$$

By introduction of PID controller we can control these above system dynamics using tuning methods and thus determine various parameters. The effects of these parameters on system response are shown in table below.

We can see that with increase in the value of k_p, we get better steady state stability as it reduces the steady-state error. The integral control can nullify the steady state error but cost paid is that it makes the system sluggish. While above two lead to oscillatory response initially, the derivative control makes the overshoot within limit and also improves the settling time.

Table 2: Effects of increasing control parameter independently

Parameter	Rise Time	Overshoot	Settling Time	Steady-State Error	Stability
k_p	Decrease	Increase	Small Change	Decrease	Degrade
k_i	Decrease	Increase	Increase	Elimination	Degrade
k_d	Minor Change	Decrease	Decrease	No effect in theory	Improve if k_d is small

This table above shows how change in various gain parameters affects the response of the system both transient and steady state.

THE 741 OPAMP

The OPAMP stands for operational amplifier. The opAmp is an amplifier with some specific important characteristics. As the word amplifier suggests, the function of an operational amplifier (op amp) is to amplify a voltage. However, the operational amplifier does much more than that. It also functions as a buffer and as a cascade which are two functions that enable simple circuits to be assembled into complex circuits to create higher level functions which are called operations $\frac{3}{4}$ hence the name operational amplifier.

Op amps have five terminals that are important. The voltage that is amplified is the difference between the voltage at the '+' terminal V_p and the voltage at the '-' terminal V_n, as shown in figure below. The amplified voltage is the output voltage V_o. Unlike the resistor and capacitor, which are both "passive" (unpowered) devices, the opamp is an "active" device. Indeed, the op amp needs a voltage supply for the amplification. The V_s^+ and the V_s^- terminals are the positive and negative supply voltages, respectively. The op amp schematic and the chip that we'll use are shown in figure below. Generally it is available in integrated chips. The pin configuration of 741 IC is as shown below:
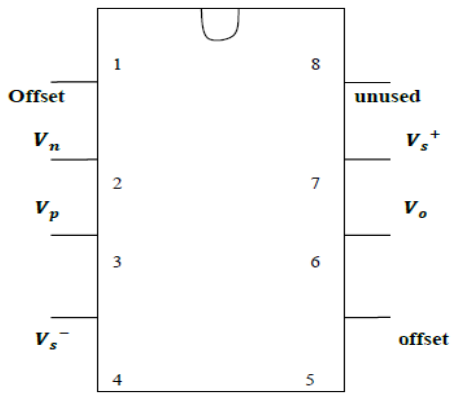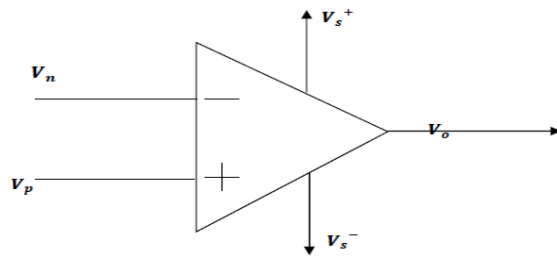
Figure 4: Pin configuration of 741 OpAmp

Figure 5: Circuit symbol of an OpAmp

DATASHEET FOR LM741
Absolute maximum Ratings
These indicate the extent which damage to the device may occur. Operating ratings indicate the circumstances for which the device is operational, but do not guarantee precise performance limits. For function at elevated temperatures, these devices must be derated based on thermal resistance.

For supply voltages less than ±15 V, the absolute maximum input voltage is equal to supply voltage.

Table 3: Datasheet for 741 IC

	LM741A	LM741	LM741C
Supply Voltage	±22 V	±22 V	±18 V
Power Dissipation	500 mW	500 mW	500 mW
Differential Input Voltage	±30 V	±30 V	±30 V
Input Voltage	±15 V	±15 V	±15 V
Output Short Circuit Duration	Continuous	Continuous	Continuous
Operating Temperature Range	-55 ^{0}C to +125 ^{0}C	-55 ^{0}C to +125 ^{0}C	-55 ^{0}C to +125 ^{0}C
Storage Temperature Range	-65 ^{0}C to +150 ^{0}C	-65 ^{0}C to +150 ^{0}C	-65 ^{0}C to +150 ^{0}C

OPAMP REALISATIONS
SIGNAL BUFFER:-
It is a circuit configuration in which input equals output. The importance of this circuit is that it isolates the input and output side. Since, the input current of opAmp is 0, loading effect is 0. So, we can measure the actual input without error due to loading. Its circuit diagram is shown below.
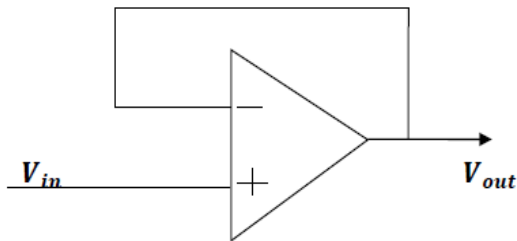

Figure 6: Signal buffer circuit

SIGNAL INVERTER:-
This circuit changes the polarity of the input signal with amplification and the gain value is,
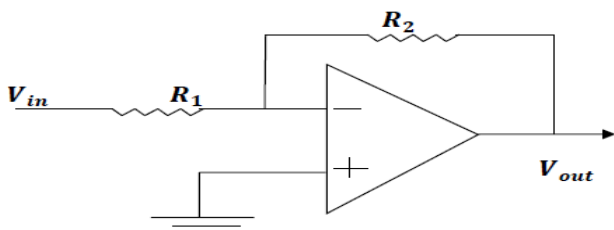
$$Gain = -\frac{R_2}{R_1}$$

Figure 7: Signal inverter circuit

SIGNAL ADDER/SUMMER:-
This circuit helps in summing up various signals .Here, output voltage is given by
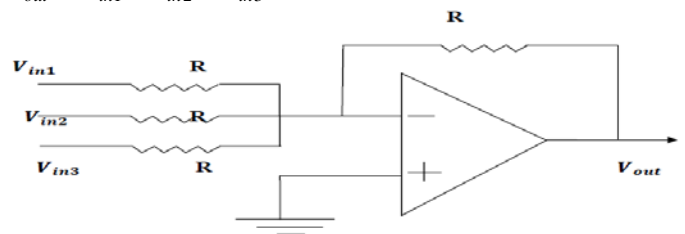
$$V_{out} = V_{in1} + V_{in2} + V_{in3}$$

Figure 8: Signal summer circuit

SIGNAL SUBTRACTOR/DIFFERENTIATOR:-
The circuit gives the difference of the two inputs given to the opamp circuit, provided all the resistances should have same value as shown in the circuit below. Here output signal is given as
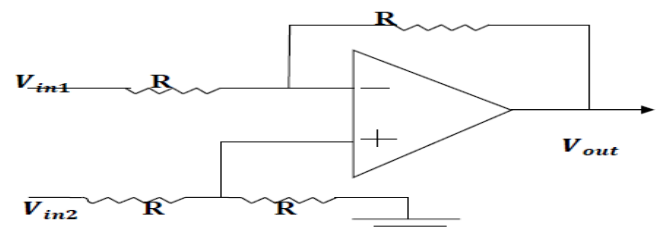
$$V_{out} = V_{in1} - V_{in2}$$

Figure 9: Signal differentiator circuit
CHOICE OF CIRCUIT PARAMETERS

The initial values of k_p, k_d and k_i for a certain PID controller are determined. Since the plant is unknown, it should be assumed that the plant is anything arbitrary and thus the controller should be tunable.

A PID controller for $0<=k_p<=100$, $0<=k_d<=10$ is needed and an arbitrary k_i as per requirement.

To begin with, there is need to test each of the controllers (proportional, integral and derivative terms) separately and then integrate them together. So, the proportional controller is assembled. As $0<=k_p<=100$, we chose our $R_2=100k$ pot and $R_1=1k$ ohms. We chose a 741 opAmp for this purpose. Initially, we set up the board as shown in circuit diagram below.
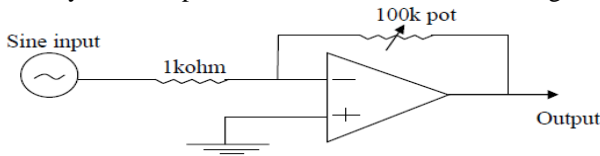
Figure 10: Proportional controller

Then, we supplied a sinusoidal voltage wave from a function generator as input to the controller circuit. The input and output waveforms were viewed in a CRO. The results were noted and waveforms were traced in tracing paper. The experiment was repeated by varying the values of k_p using potentiometer. Results were viewed and traced.

Now, we needed to do the same test with the derivative controller. Here, we needed to supply a ramp input and check the output. Since, ramp signal cannot be generated due to saturation, so, we used a triangular wave input to the controller.

As we required $0<=k_d<=10$, we use a 10 micro Farad capacitor, a 1K resistor and a 1M pot for the purpose, as shown in below circuit diagram. Waveforms were viewed in CRO and traced in tracing paper.
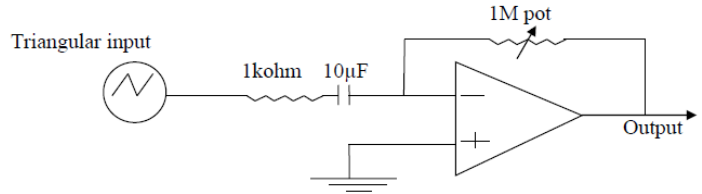

Figure 11: Derivative controller

Next, we repeated the test for integral controller with circuit diagram as shown below. Components required were 1 micro Farad capacitor and 1M pot and a small, resistance say 1kohm was put in series with the capacitor as shown in the circuit diagram above.

Input given to the controller was a square wave. Output waveforms were viewed and traced in a tracing paper. Results were obtained for different values of by k_i varying the potentiometer. The same was repeated by replacing the 1 micro Farad capacitor with a 10 micro Farad capacitor.
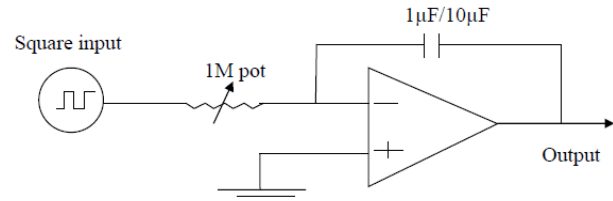

Figure 12: Integral controller

Now, after performing these entire tests we move on to fabricate our PID controller .The circuit diagram for the design is shown below. The components used for whole process are shown in table below. The components are assembled together and the connections were made as per circuit diagram on the bread board. The supply voltages for the 741 opamps are not shown in the circuit diagram. Supply voltage of ±15 V was given to the IC's. Then inputs were supplied using function generator and the required waveforms were traced on the tracing paper. Finally, after the testing the components were removed from the bread board and fabrication was started. Thus, fabrication of PID controller was completed.
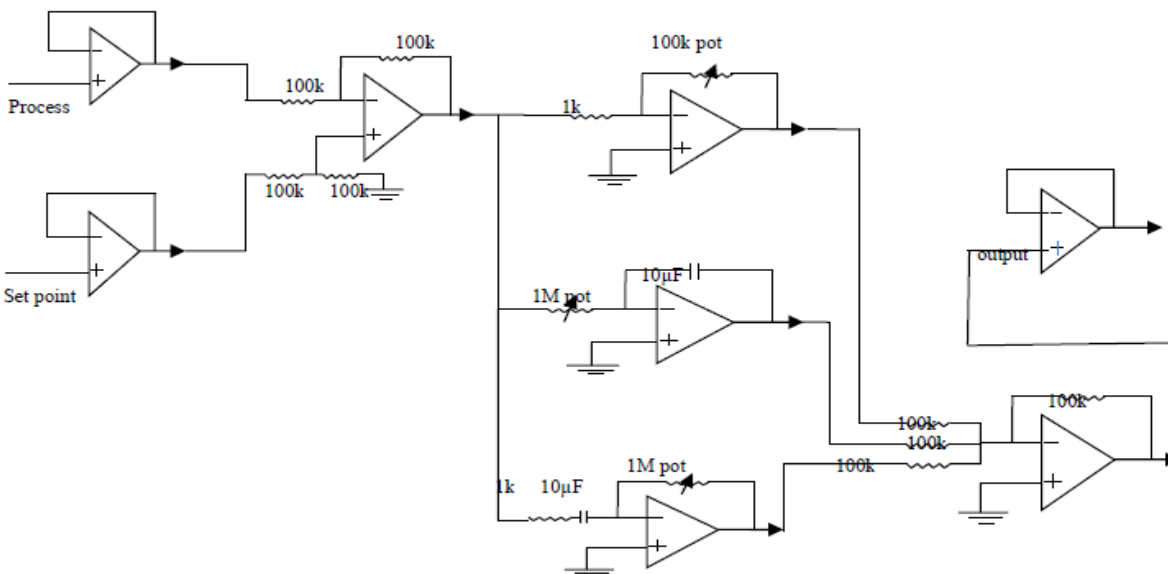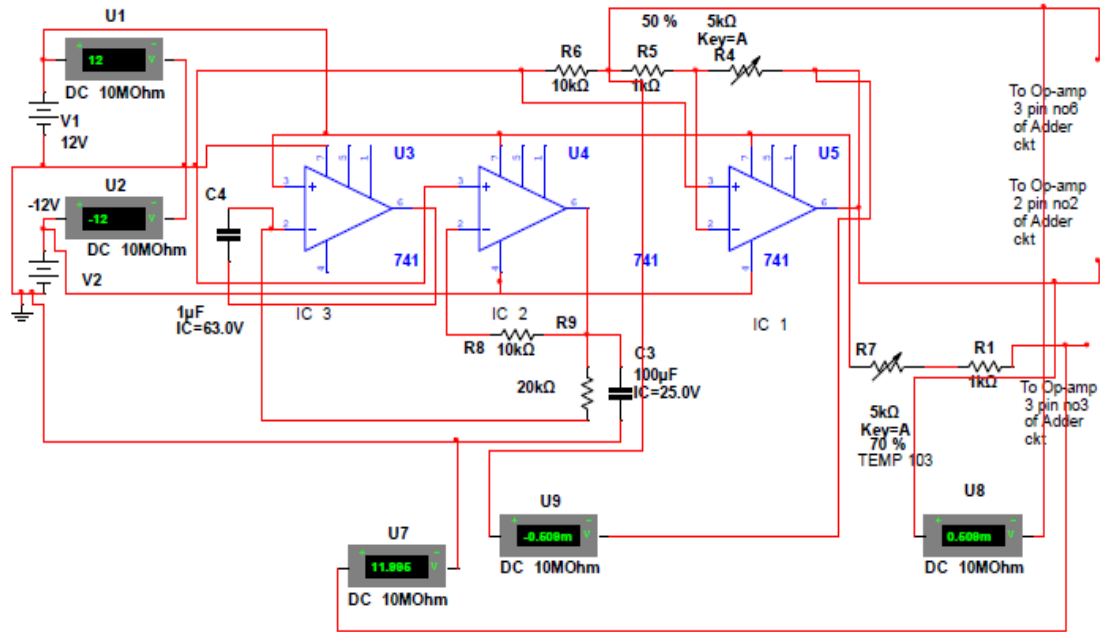

Figure 13: Complete PID circuit

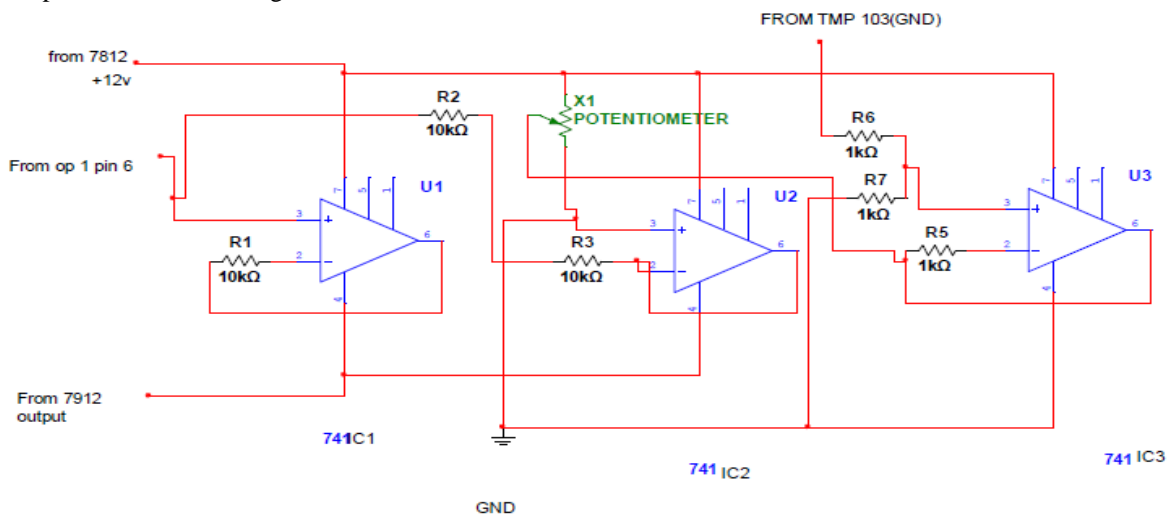**Figure 14:** Complete PID circuit using Multisim

Figure 15: Adder/Inverter

Dedicated as input buffer, while one for output buffer. The process and set point variables were given at the input and same values of input at the output terminals were obtained. Subsequently, both the inputs are subtracted using another opamp IC which utilizes four equivalent resistors of 100k each. This produces an error signal at its output. The output response of this is given to each of the specific controller viz., proportional, integral and derivative. The controllers are three signal inverters with two resistors in proportional control and one capacitor and one resistor in both integral and derivative controls with their position switched in each. The output of the three controllers are collated together using a summer circuit and then passed through a buffer circuit. Using the buffer circuit, the whole control circuit is isolated from outside loads. Three potentiometers were used to realize the control variables as shown in figure 15. As observed, the proportional term contains a 100M pot, while derivative and integral terms have 100M pots,

executed in order to attain required range of values of k_p, k_i, and k_d.

The derivative control shows a small resistor of 1k. This is presented in order to save the capacitor from short circuiting. Thus, this resistor limits the short circuit current.

Table 4: Components used in fabricating PID controller

S/No	Components used	Quantity
1	OpAmps (741)	8
2	100k pot	1
3	1M pot	2
4	100k resistors	8
5	1k resistor	2
6	1microFarad capacitor	1
7	10microFarad capacitor	2
8	Soldering kit	-
9	Multimeter	-

10	CRO	-
11	±18v power supply	-
12	Connecting wires	As required
13	Bread board/proto board	-
14	7815	2
15	Fan to fan connecter	As required
16	Berg strip	As required

TEST RESULTS

The principal technique refers to the test stand and test strategy, utilizing an signal function generator and a digital oscilloscope.

In the initial step, the accuracy of every control law of the PID controller will be independently demonstrated by applying a train of rectangular sign to the proportional, derivative and integral component. The acquired results are appeared in figures 16 - 18 below. From those three outlines, the particular amplifying, derivative and integrative impact acknowledged by the PID controller can be seen.
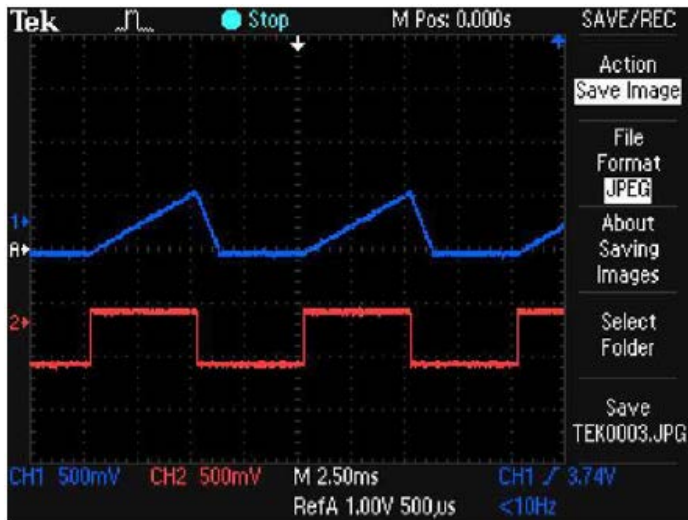

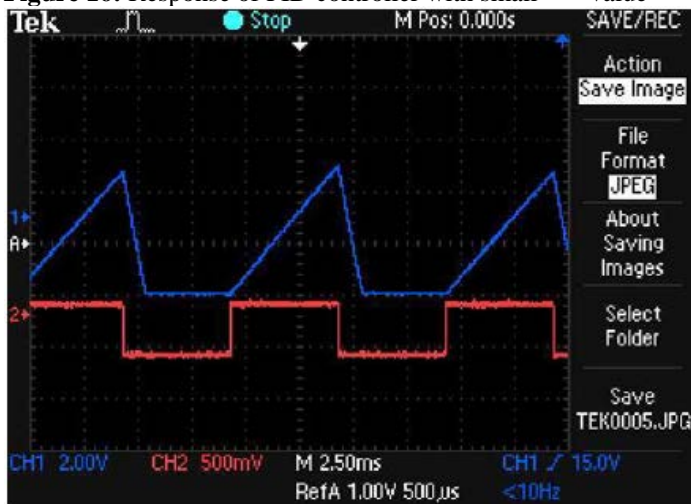**Figure 16:** Response of PID controller with small k_p value

Figure 17: Response of PID controller with large k_p value

The source voltage given through an adapter was ±18 V which was then converted to ±15 V using 7815. Then required test was performed on proportional controller, it is observed that the peak to peak value of sine waveform got reduced with increase in the value of k_p, and also, the waveform approached a steady dc value with nearly no ripples.
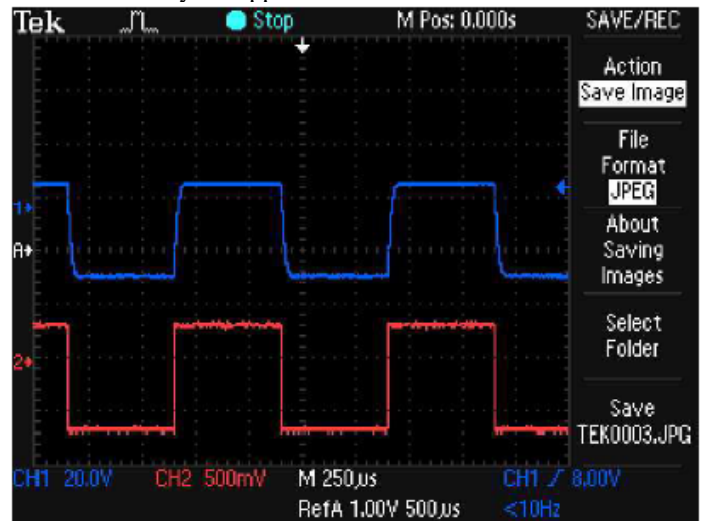

Figure 18: Output response of the PID controller with k_p value and $k_d = k_i = 0$

The output response of the derivative controller shows a square wave equivalent to a triangular input as estimated. The variation of k_d had no basic effect on the waveform except that there was a minor variation in duty cycle for variation of k_d from 0 to 10.

It is again observed that the output response of the integral controller showed both positive and negative peaks when a 1 micro Farad capacitor was used. When the capacitor was replaced by a 10 micro Farad capacitor the output waveform was similar to input wave with a large rise and decay time.

CONCLUSION

The response of the controller can be classified as:

i.   the responsiveness of the controller to an error,

ii. the degree to which the controller overshoots the set point and the system oscillation.

The PID controller offers the opportunity to act with numerous control law.

It was observed that the proportional controller reduces the transients to a significant extent and thus, k_p should have high value. The derivative controller acts on the rate of change of input and thus converts the triangular wave to a square wave. It is very sensitive to changes or variations in the input.

As for the integral controller concerned, it shows a slow rise and decay time making system sluggish. The output waveforms were found suitable as expected. Consequently, the analog PID controller was fabricated.

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