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Simulation of Harmonic motion

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ABSTRACT

In this work, we study the characteristics of simple harmonic motion and to solve physical problems related to simple harmonic motion (SHM) using MATLAB computer program. To achieve the objectives, the necessary techniques had been done using both analytical and numerical mathematical computations. Among the numerical methods, we had concentrate Runge Kutta forth order method. The solutions would be given by figures. For the undamped and damped SHM we solve numerically using the above numerical methods with applying the differential algorithm. With this respect, by adjusting the set of equidistant points (the step size) the numerical solution is comparable with the analytical one. For both cases, the undamped and damped SHM the numerical solution obtained using Runge-kutta is almost the same as to the solution obtained using analytical method.

Keywords: *simple harmonic motion, Runge kutta fourth, analytical solution ,numerical solution*

INTRODUCTION

Oscillation is a periodic fluctuation in the value of a physical quantity above and below some central equilibrium value. Any motion or event that repeats itself at regular intervals is said to be periodic motion. In some periodic motions a body moves back and forth along a given path between two extreme positions. The backward and forward swing of a pendulum, the up and down motion of a weight hanging on a spring, and the twisting and untwisting motion of a body suspended by a wire are the examples of motions which are simple harmonic. Such periodic motions are called oscillatory motions [2].

In this work I discussed only mechanical oscillations, but the techniques I developed, applicable to other kinds of oscillatory behavior. A special kind of periodic motion occurs when the force that acts on a particle is proportional to the displacement of the particle from the equilibrium position and always directed toward the equilibrium position. When this type of force acts on a particle, the particle exhibits simple harmonic motion (SHM), which will serve as an analysis model for a large class of oscillation problems. If friction or some other mechanism, causes the energy to decrease, the oscillation sis said to be damped SHM [3]. For our purpose we focused on the numerical and analytical computation of the physical problem. For the numerical one we prefer Runge Kutta fourth and for the analytical one we prefer the analytical solution of SHM in the absence of damping force and in the presence of damping force and also we compare the numerical with the analytical solution.

1.1 Simple harmonic motion (SHM)

Simple harmonic motion is a motion that is taking place in which the acceleration of an object is proportional to displacement of the object from a fixed point and in opposite direction to the displacement. For SHM to occur, three conditions must be satisfied: first there must be a stable equilibrium position, second, there must be no dissipation of energy, third, the acceleration must be proportional to the displacement and opposite in direction [2][4]. A typical system that exhibits SHM is an object suspended on a spring as shown in Figure 1. In equilibrium the spring exerts no force there is a force on the body (mass) which tends always to store it to the equilibrium position. And we call this a restoring force ($\boldsymbol{F_r}$). As we can see from Figure 1, **F** and **y** always have opposite signs. The force on a stretched or compressed spring is given as $F_{app} = ky$. The force of the spring exerts on the body is the negative of the applied force. So that the spring force along **y** axis

$$F_r = -ky \quad (1)$$

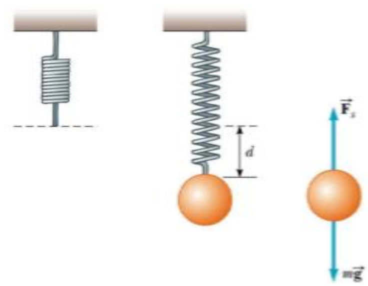

Figure 1: Idealized model undamped SHM adopted from Halliday (1984)

The only force that accelerates is the spring (restoring) force. Thus,

$$\sum F = F_r \quad (2)$$

$$m\vec{a} = -ky \quad (3)$$

$$\vec{a} = -\frac{k}{m}y \quad (4)$$

This acceleration is not uniform. So we can't use equation of uniform motion. This acceleration is proportional to the displacement **y** but opposite to each other. That is, when **y** is positive $\vec{a}$ is negative and vice versa. This is the characteristic of SHM.

1.2 Equation of SHM in the absence of damped force

From equation (4), we have;

$$\vec{a} = -\frac{k}{m}y \quad (5)$$

and from the definition of acceleration, we have the angular frequency of SHM. Then the differential equation of simple harmonic motion in the absence of damping force becomes;

$$\frac{d^2y}{dt^2} - w_0^2 y \qquad 6$$

Equation (6) is linear, second order and homogeneous ordinary differential equation (ODE). And this differential equation can be treated by means of two separate methods, analytically and numerically.

1.3 Analytical solution of SHM in the absence of damping force

The general equation of second order ordinary differential equation in the form of

$$\frac{d^2y}{dt^2} + a\frac{dy}{dt} + by = (D^2 + aD + b)y = 0 \qquad 7$$

Hence, the analytical solution of undamped simple harmonic motion is

$$y(t) = A\cos(\omega_0 t + \phi) \qquad 8$$

Analytical solution of in SHM the presence of damping force

For damping SHM, the auxiliary equation is:

$$p^2 + \frac{b}{m}p + \frac{k}{m} = 0 \qquad 9$$

With roots

$$p_1 = \frac{-b}{2m} + \frac{1}{\sqrt{b^2 - 4km}}$$

$$p_2 = \frac{-b}{2m} - \frac{1}{\sqrt{b^2 - 4km}}$$

and

Thus the solution is given as

$$y(t) = c_1\exp(p_1 t) + c_2\exp(p_2 t) \qquad 10$$

Let us see some special cases for the equation of damped SHM. That is as the term $b^2 = 4km$ is less than, greater than or equal to zero. We now have three cases, resulting in quit different motion of the oscillator. If $b^2 = 4km > 0$ (over damping), $b^2 = 4km < 0$, (under damped) and $b^2 = 4km = 0$ (Critical damping).

and the solution has the form:

$$y(t) = C\exp(-b\frac{t}{2m})\cos(\omega t - \phi)$$

Where, C the amplitude of damped SHM ϕ phase constant and $\omega = \sqrt{\frac{4k - b^2}{2m}}$ angular frequency of the damped SHM.

1.4 Numerical solution of equation of SHM

Solutions to ordinary differential equations of SHM in the above are expressed in terms of mathematical functions such as the sine, cosine and exponential. These functions are analytical and the solution can be found to any degree of accuracy. Another technique which has gained in importance since the advent of high speed and large memory computers is the numerical techniques. There are different methods which help to solve numerically such kind of equations. We want to solve the following equation numerically.

$$\frac{d^2y}{dt^2} + w_0^2 y = 0 \qquad 11$$

To solve equation (11) numerically, first we have to change into first order differential equation
in the following way.

$$\frac{d}{dt}(\frac{dy}{dt}) = -w_0^2 y \qquad 12$$

Let $\frac{dy}{dt} = v = f(y,t).$ becomes

$$\frac{d}{dt}(\frac{dv}{dt}) = -w_0^2 y = g(v,y,t) \qquad 13$$

Using the same producer doing in the equation of undamped SHM we can change equation
of the damped SHM into first order as follow:

$$(\frac{dv}{dt}) = -\frac{b}{m}v - \frac{k}{m}y = g(v,y,t) \qquad 14$$

1.5 Runge kutta fourth order method

The Runge-kutta forth order method is probably the most extensively used method for solving the initial value problem of general second order differential equations. We have used the MATLAB software for carrying out the calculations; According to Runge-Kutta forth ordermethod, the solution to equations (13) and (14) are given by:

$$y(i+1) = y(i) + \frac{1}{6}(k_1 + 2k_2 + 3k_3 + k_4) \qquad 15$$

And

$$v(i+1) = v(i) + \frac{1}{6}(m_1 + 2m_2 + 3m_3 + k_4) \qquad 16$$

Here, $y(i)$ and $v(i)$ represents, the values of $y(t)$ and $v(t)$ respectively . at $t = t(i)$. Furthermore, the values of the variables k_i and m_i ($i = 1,2,3,4$) are given as follows:

$$K_1 = hv(i) \qquad 17$$

$$K_2 = h(v(i) + \frac{m_1}{2}) \qquad 18$$

$$K_3 = h(v(i) + \frac{m_2}{2}) \qquad 19$$

$$K_4 = h(v(i) + m_3) \qquad 20$$

$$m_1 = hf(t,y,v), \qquad 21$$

$$m_1 = hf(t,y,v), \qquad 22$$

$$m_2 = hf(t + \frac{h}{2}, y + \frac{k_1}{2}, v + \frac{m_1}{2}), \qquad 25$$

$$m_3 = hf(t + \frac{h}{2}, y + \frac{k_2}{2}, v + \frac{m_2}{2}) \qquad 26$$

$$m_4 = hf(t + h, y + k_3, v + m_3) \qquad 27$$

RESULT AND DISCUSSION

2.1 Simulation of undamped SHM using analytical method

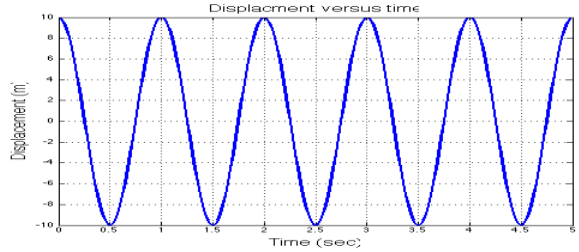

Figure 2 Simulation of undamped SHM using analytical method

Here, figure 1 is the graph of displacement versus time of ideal SHM. It shows that the exact behavior of undamped SHM, that is, the amplitude doesn't depend on time it remains constant with time. Furthermore, the oscillation is started its rotation from the positive maximum displacement. This is due to the phase angle. In this case the phase angle is zero, but if the phase angle is $\frac{\pi}{2}$, the oscillation starts its rotation from the origin and if it is π , the rotation starts from the negative maximum displacement.

2.2 Simulation of undamped SHM using Runge-kutta fourth order method

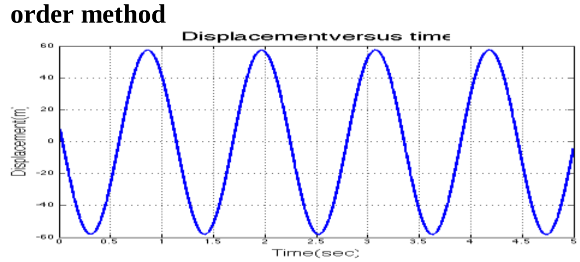

Figure 3: Simulation of undamped SHM using Runge-kutta fourth order method

In this section, we will demonstrate simulation of the above problem, which is a result of MATLAB program using Runge-Kutta forth order mathematical computation and results the following graph. When we see Figure 2, the oscillations are seem to be quite stable, that is, their amplitude remains constant with time, just as the result we obtained in the exact solution. Thus, Runge-Kutta method is fitted with the exact solution.

2.3 Simulation undamped SHM using Runge-kutta fourth order method and analytical method

As shown in Figure 3, which is the result of numerical and analytical solution of equation of undamped SHM, the runge kutta fourth order method is closer to the exact solution. We have seen the graph the time increase the error increase. The blue color shows that range kutta fourth order method and the green color shows that the analytical solution. I have detail MATLAB program (script), which can be used to compare this numerical and analytical solution.

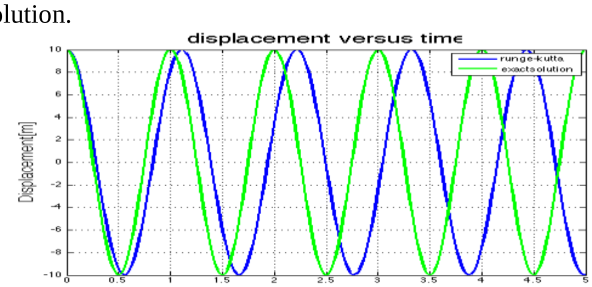

Figure 4 Simulation of undamped SHM using Runge-kutta fourth order method and analytical method

2.4 **Simulation damped SHM analytical method**

Figure 4 is a result of the problem when the motion is inside a water with damped parameter $z = \frac{b}{2m} = 3.58$. The solution of this problem is solved using MATLAB program (see appendix D) using analytical mathematical computation and results the following graph

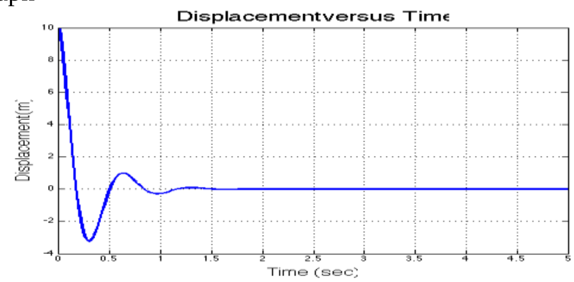

Figure 5: Simulation of damped SHM analytical method

The simulation shown in figure 5, shows the system does not indeed oscillate. That is the amplitude of the oscillation appears to decrease with time and becomes zero after some time.

2.5 Simulation damped SHM using Runge-kutta fourth order method

The result in Figure 6, which demonstrates simulation of damped simple harmonic motion which is a result of MATLAB program using Runge-Kutta fourth order method. From this figure we can see that the solution obtained by Runge-Kutta forth order method is close to the exact solution, Thus for the same value of h (step size) the Runge Kutta forth method appears to give accurate result.

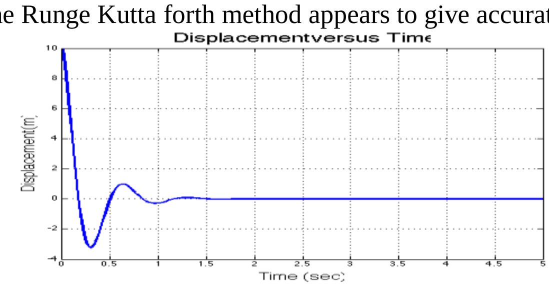

Figure 6: Simulation of damped SHM using Runge-kutta fourth order method

2.6 Simulation damped SHM using Runge-kutta fourth order method and analytical method

As shown in Figure 7, which is the result of numerical and analytical solution of equation of damped SHM, the Runge-kutta fourth order method is close to the exact solution The detail MATLAB program, which can be used to compare this numerical and analytical solution is cited at the back of this paper in Appendix F. We have seen the graph the time increase the error decrease, because of the oscillation die out through time.

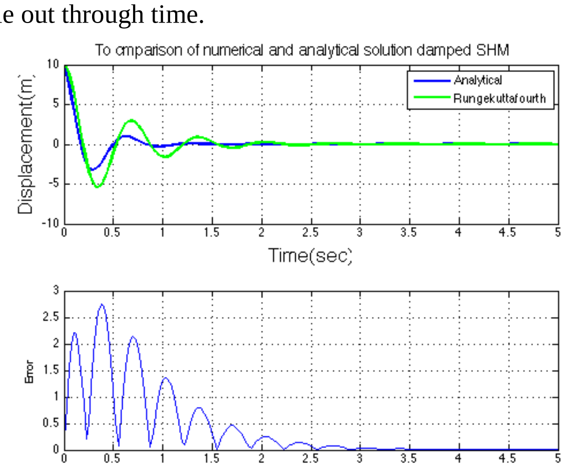

Figure7: Simulation of damped SHM using Runge-kutta fourth order method and analytical Method

2.7 Simulation of damped oscillatory motion for three cases of damped SHM.

As we can see from Figure 9 the over damped and the critical damped cannot give an oscillatory motion just as expected. That indicates the damed parameter increase the oscillation decrease.

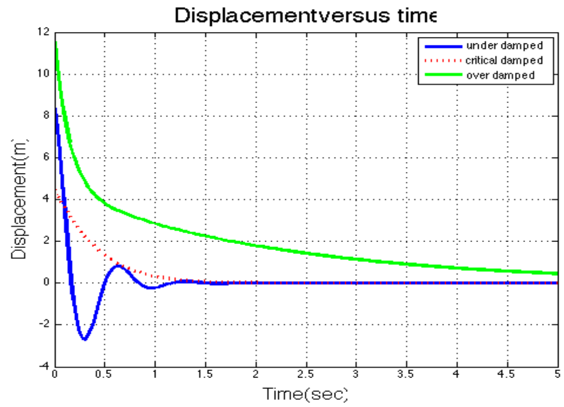

Figure 8: Simulation of damped oscillatory motion for three cases of damped SHM.

CONCLUSION

Generally, in this work to understand the behavior of simple harmonic motion and to apply and solve physical problems related to SHM using numerical analysis by using MATLAB computer program. And we found the solution of damped and undamped SHM using analytical and numerical methods. For the numerical model use Runge-Kutta fourth order method, when we use Runge-kutta fourth order method is same as the exact solution for the damped and undamped SHM. From this we conclude that Runge-kutta fourth method is the best to solve for our problem numerically. In this project we have considered several different methods for dealing with ordinary differential equations of SHM for which initial conditions are specified and have estimated the numerical errors associated with each.

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