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Effect of Denoising Dependent and Independent Variables on the Performances of Non-Linear Regression Parameters of the Estimators

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ABSTRACT
Denosing is any signal processing method which reconstructs a signal from a noisy one. Its goal is to remove noise and preserve useful information. In the study simulated data under three (3) sample sizes (i.e 32,256 and 1024) were used, applying Epanechnikov kernel Gaussian kernel, Wavelet and Polynomial Spline to denoise the variables in two approaches. The study revealed the performances of denoised nonlinear estimators under different sample sizes and comparison was made using the mean squared error criterion. The result of the studies showed that the approach of denoising both the dependent and independent variables enhance the performances of non-linear least squares estimator (DNLS) under each sample size for the different four smoothers considered.
Keywords: *production function, Monte-Carlo Simulation, Denoising, Measurement error, Smoothers, Non-linear regression model, Dependent variable and Independent variables.*

INTRODUCTION
The essence of regression in econometrics is to generalized for the population from what we derive from the sample. For instance, the linear relationship from

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \ldots \beta_k X_{kt} + u_t$$

holds for the population only if we could obtain conceivable value of X's and Y which form the population value of the variables. Since this is impossible, in practice, the alternative is to get sample observation values for X's and Y specify the distribution of the errors (u_t) and try to get satisfactory estimates of true parameters of the relationship.

Non-linear regression model is one for which the first-order conditions for least squares estimation of the parameters are non-linear function of the parameters. The data consist of independent variables (explanatory variables) and their associated observed dependent variables (response variables) which may contain measurement error or noise. Variables are said to be noisy if they are not exactly equal to the variable of interest because the generating system of the measurement may not be perfectly measured. In statistics, an error is not a mistake because variability is an inherent part of things being measured and of the measurement process.

Since the observed data from various fields are frequently characterized by measurement errors, to construct a satisfactory unbiased and consistent estimator of the parameters in a non linear regression has been a challenging problem, which leads to various degrees of erroneous conclusions in economic analysis. Ordinary least square (OLS) estimators are most frequently used for simple regression models and known to be consistent. However, both dependent and independent variables may contain measurement errors as a result of this, OLS estimators are inconsistent (see carroll,etal 1995) and cheng and vanness (1999,pp.3,11))[1,2] Since the least square estimator is not consistent because of the measurement errors.

To overcome this problem of measurement errors, a natural approach, called smoothing techniques is applied to handle the noisy data for proper removal of the noisy observation (i.e denoise the data). In statistics to smoothen a data is to create an approximating function that attempt to capture the important pattern in the data, while leaving out noise or other fine scale structure or random phenomena. There are several method of smoothing techniques which can be used to extracts more information from data in as much the assumption of the smoothing is reasonable and provides flexible and robust analysis, such as: wavelets, others are kernels and polynomial spline e.t.c.

There have been many studies on denoising. So far, denoising has been extended to least squares estimator, least absolute deviation estimator and M-estimator using kernel, wavelet and polynomial spline as smoothers.The study conducted by Cai,et al (2000) proposed a solution to the problem of controlling measurement error. They used wavelet to denoise both dependent and independent variables, and fit a linear regression model and resulting estimators called denoised least squares estimators (DLS). They studied its consistency and applied to the area of marketing science. Their results showed that DLS outperform OLS. In related study, cui and Hu (2010)[3] proposed a particular method of denoised non-linear regression using a kernel-type smoothing for independent variables. The bias and the standard errors of the proposed estimators like the denoised non-linear least squares (DNLS), denoised non-linear absolute deviation (DNLAD) and non-linear huber's M (DNM) estimators. Cui and Hu (2010)[3] carried out a simulation, showing comparison among the DNLS, DNLAD and DNM. It was observed that the denoised non-linear regression estimation show a good performance.

In recent studies, a natural class of denoised non-linear regression estimators has been used for the estimation of nonlinear error model. Fasoranbaku and Soyombo (2015)[4] employed the Epanechnikov, Gaussian, wavelet and polynomial spline smoothers, the performance of denoised nonlinear least square estimators (DNLS), denoised nonlinear least absolute deviation (DNLAD) and denoised nonlinear M-estimator (DNM) are compared based on mean square error (MSE) criterion to determine their efficiency. The simulation studies carried out for n=1024 with 1000 Monte Carlo samples, show that the denoised nonlinear least squared has the smallest MSE under the four smoothers considered. Soyombo and Fasoranbaku (2015)[6] also used the known Epanechnikov Kernel smoother,to perform the denoising procedures, carry out simulation studies under some settings to determine the performance of the denoised non-linear estimators when the parameter values are varied. The results show that the DNLS outperforms both the DNLAD and DNM. Therefore, parameters of non-linear model are not sensitive and thus have no effect on the performance of denoised non-linear estimators. Fasoranbaku and Alabi (2016) [5] conducted a simulation study under three different sample sizes (32, 256 and 1024), applied earlier mentioned smoothers to denoise the only dependent variables to determine the effect of sample size on performances of non-linear estimators and their results showed that DNLS performed better under the three sample sizes but the large sample size (1024) enhance the performance better.

For the purpose of estimating the error model, this study investigates Cobb Douglas production model in economics. The function with additive error is written as:

$$P_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} + u_t \qquad (1.1)$$

$$(\beta_1 > 0), \quad (0 < \beta_2 < 1), \quad (0 < \beta_3 < 1)$$

where P_t is output at time t (the real value of all goods produced in a period of time)

L_t is the Labour input (the number of person hours in a period of time), K_t is the Capital input (the real value of Machinery and Building), β_1 is a Constant, (total factor productivity), β_2 and β_3 are the output elasticity of Labour and Capital (measure the respective contribution of L_t and K_t to the production process) and u_t is the stochastic disturbance term

Suppose that $\{(L_t, K_t, P_t): 1 \le t \le n\}$ are unobservable "true" variables satisfying a nonlinear relationship, measurements of (L_t, K_t, P_t) are collected to yield an observable data set of $\{(x_{t1}, x_{t2}, y_t): 1 \le t \le n\}$ i.e. the true variables plus additive measurement errors such that

$$x_{t1} = L_t + \delta_t, \quad x_{t2} = K_t + \varepsilon_t \text{ and } y_t = P_t + u_t \qquad (1.2)$$

where δ_t and ε_t are measurement errors. To be in line with the usual nonlinear model, the model (1.1) becomes:

$$y_t = \beta_1 x_{t1}^{\beta_2} x_{t2}^{\beta_3} + u_t \qquad (1.3)$$

In estimating denoised non-linear regression parameters of the estimators, the effect of denoising both the dependent and independent variables approach have not been extensively experimented. Though Cui and Hu (2010)[3] conducted small simulation study to compare the performances between the approach to denoise both dependent and independent variables and to denoise only the independent variables. According to their limited experiments they found that: denoising the dependent variable can enhance the performance of the estimator in some cases but the enhancement may be limited especially as sample size increases: in some other cases the story may be difference. Therefore, this study would employ three different denoised estimators (DNLS, DNLAD, and DNM) to estimate non-linear regression parameters of the estimators under three different sample sizes (32, 256, and 1024). The goal of the study is to examine whether the approach to denoise both independent and dependent variables enhance the performance of the nonlinear estimators in estimating non-linear regression parameters. The article is arranged as follows. Section 2, specifically considered two family of kernel smoothing, wavelet smoothing and polynomial spline smoothing procedures. Section 3 shows the linearization of non-linear Cobb Douglas production model. Section 4 defines DNLS, DNLAD and DNM. Section 5 report a Monte-Carlo study to compare the performance of DNLS, DNLAD and DNM estimators under the three sample sizes by using the earlier mentioned smoothers to denoised only the explanatory variables.

2. Denoising Procedures

The basic idea behind smoothing a data set is the creation of an approximating function that attempts to capture important patterns in the data while leaving out the noise, and is also referred to as "denoising". There are various methods to help restore a data set from measurement noise. In this study, the following smoothing method are used

1) Kernel smoothing: Given a random sample $X_1 \ldots X_n$ with a continuous, univariate density function $f(.)$,The kernel density estimator is:

$$\hat{f}(x,h) = \frac{1}{nh}\sum_{i=1}^{n} k\left(\frac{x - X_i}{h}\right) \qquad (2.1)$$

Where x is the value of the scalar variable for which one seeks an estimate while X_i are the values of that variable in the data. K is a function of a single variable called the *kernel*. The kernel determines the *shape* of the function. The parameter *h* is called the *bandwidth* or *smoothing constant*. It controls the degree of smoothing and adjusts the size and form of the function.

$$u = \left(\frac{x - X_i}{h}\right) \qquad (2.2)$$

For the purpose of this study, the two most commonly used Kernels function are utilized:

a) Epanechnikov Kernel smoothing:

$$K(u) = 0.75(1 - u^2) I_{(|x| \le 1)} \; on \quad u \in (-1,1) \qquad (2.3)$$

b) Gaussian Kernel smoothing:

$$k(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) \qquad (2.4)$$

2) Polynomial spline denoising: A smoothing spline is a method of smoothing (fitting a smooth curve to a set of noisy observations) using a spline function which minimizes:

$$p(y,x) = \sum [y_i - \hat{u}(x_i)]^2 \lambda \int_{x_1}^{x_n} \hat{u}''(x)^2 dx \qquad (2.5)$$

where λ is positive smoothing parameter which controls the amount of smoothing of the data, it is defined between 0 and 1. $\lambda = 0$ Produces least squares straight line fit to the data, while $\lambda = 1$ produces a piecewise cubic polynomial fit that passes through the data points.

3) Wavelet denoising: they are generated from dilations and translations of a "father" wavelet ϕ

$$\varphi_{j_0,k}(x) = 2^{\frac{j_0}{2}} \phi(2^{j_0} x - k); k = 0,1,\ldots,2^{j_0} - 1$$

(2.6)

and a “mother” wavelet ψ .

$$\psi_{j,k}(x)=2^{\frac{j}{2}}\psi(2^{j}x-k);\ j=j_0,...,j;k=0,1,...,2^{j}-1$$

(2.7)

3. Linearization of Non-linear Function using Newton Raphson Approximation Method

Let us consider (1.1)

$$P_t=\beta_1 L_t^{\beta_2}K_t^{\beta_3}+u_t \quad (3.1)$$

$$g(\beta)\approx g(\beta^t)+G(\beta^t)(\beta-\beta^t)+\frac{1}{2}(\beta-\beta^t)'H(\beta^t)(\beta-\beta^t) \quad (3.2)$$

Where, $G(\beta^t)=\left[\frac{\partial g}{\partial \beta_i}\right]_{\beta^t}$ is the score vector and $H(\beta^t)=\left[\frac{\partial^2 g}{\partial\beta_i\partial\beta_k}\right]_{\beta^t}$ is the Hessian matrix.

$$\sum_{t=1}^{n}u^2=\sum[p_t-f(\beta_1,\beta_2,\beta_3)]^2=S(\beta) \quad (3.3)$$

$$G(\beta\ )=\left[\frac{\partial S(\beta)}{\partial\beta_1},\frac{\partial S(\beta)}{\partial\beta_2},\frac{\partial S(\beta)}{\partial\beta_3}\right]'$$

$$H(\beta\ )=\begin{bmatrix}\frac{\partial^2 S(\beta)}{\partial\beta^2_1},\frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_2},\frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_3}\\ \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_2},\frac{\partial^2 S(\beta)}{\partial\beta_2^2},\frac{\partial^2 S(\beta)}{\partial\beta_2\partial\beta_3}\\ \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_3},\frac{\partial^2 S(\beta)}{\partial\beta_2\partial\beta_3},\frac{\partial^2 S(\beta)}{\partial\beta_3^2}\end{bmatrix}$$

This Hessian matrix is positive definite, the maximum of the approximation of $g(\beta)$ occurs when its derivative is zero

$$G(\beta^t)+H(\beta^t)(\beta-\beta^t)=0 \quad (3.4)$$

$$\beta=\beta^t-\left[H(\beta^t)\right]^{-1}G(\beta^t) \quad (3.5)$$

This gives a way to compute β^{t+1}, the next value in iterations, and is defined as

$$\beta^{t+1}=\beta^t-\left[H(\beta^t)\right]^{-1}G(\beta^t) \quad (3.6)$$

The iteration procedures continue until convergence is achieved. Near the maximum the rate of convergence is quadratic as defined by $\left|\beta^{t+1}-\hat{\beta}_t\right|\le c\left|\beta^t-\hat{\beta}_t\right|^2$ for some $c\ge 0$ when β_i^t is near $\hat{\beta}_t$ for all i. Thus we get estimates β_i^t by Newton Raphson methods.

From the linearization result in equation (3.5) we can obtain estimate of β_1,β_2,β_3 as follow:

$$\begin{bmatrix}\beta_1\\ \beta_2\\ \beta_3\end{bmatrix}=\begin{bmatrix}\beta_1^0\\ \beta_2^0\\ \beta_3^0\end{bmatrix}-\begin{bmatrix}\frac{\partial^2 S(\beta)}{\partial\beta^2_1},\frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_2},\frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_3}\\ \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_2},\frac{\partial^2 S(\beta)}{\partial\beta_2^2},\frac{\partial^2 S(\beta)}{\partial\beta_2\partial\beta_3}\\ \frac{\partial^2 S(\beta)}{\partial\beta_1\partial\beta_3},\frac{\partial^2 S(\beta)}{\partial\beta_2\partial\beta_3},\frac{\partial^2 S(\beta)}{\partial\beta_3^2}\end{bmatrix}^{-1}\begin{bmatrix}\frac{\partial S(\beta)}{\partial\beta_1}\\ \frac{\partial S(\beta)}{\partial\beta_2}\\ \frac{\partial S(\beta)}{\partial\beta_3}\end{bmatrix}$$

Once a parameter vector is obtained, the estimates are likely better than the old trial estimates, and so can be used in place of $(\beta_1^0,\beta_2^0,\beta_3^0)$ and the computation can be done again. The iteration can continue, obtaining new and better estimates until the difference between successive parameter vectors is small enough to assume convergence.

4. **Denoised Non-Linear Regression Estimators.**

When the regressors in a non-linear regression model are subject to measurement errors, it becomes a problem to construct consistent estimators of the parameters. It is possible, however, to construct consistent estimators in a non-linear model like (1.1) by first applying the denoising techniques discussed ealier to the variables, then estimators like the least squares, least absolute deviation and M-estimator will be applied to these denoised variables to yield consistent estimators which are called

i. Denoised nonlinear least squares (DNLS) of $(\beta_1,\beta_2,\beta_3)$ minimizes

$$D_n=\sum_{t=1}^{n}[P_t-f(\hat{L}_t,\hat{K}_t,\beta_i)]^2 \quad i=1,2,3 \quad (4.1)$$

ii. Denoised nonlinear least absolute deviation(DNLAD) of $(\beta_1,\beta_2,\beta_3)$ minimizes

$$L_n=\arg\min_{\beta_i}\sum_{t=1}^{n}\left|P_t-f(\hat{L}_t,\hat{K}_t,\beta_i)\right| \quad (4.2)$$

where β_i is the solution of the parameters and

iii. Denoised M-estimators

$$M_n=\arg\min_{\beta_i}\sum_{t=1}^{n}\rho\left[P_t-f(\hat{L}_t,\hat{K}_t,\beta_i)\right] \quad (4.3)$$

Where ρ is a loss function. The function ρ can be chosen in such a way to provide desirable properties of estimators (in terms of bias and efficiency) when the data are truly from the assumed distribution. Least-squares estimators are special M-estimators with $\rho(x)=x^2$, where $x=\left[P_t-f(\hat{L}_t,\hat{K}_t,\beta_i)\right]$

5. Simulation Studies

A Monte Carlo simulation is a problem solving technique used to approximate the probability of certain outcomes by running multiple trials, using random variables.

In this work, an extensive Monte Carlo simulation is conducted to generate random data of sample size 32, 256 and 1024 to examine the performance of the denoised nonlinear estimators from the model.

$$y_t=P_t+u_t \quad and \quad x_{t1}=L_t+\delta_t, \quad x_{t2}=K_t+\varepsilon_t \quad (4.1)$$

where $\hat{L}_t\sim U(1,30)$, $\hat{K}_t\sim U(10,200)$, $u_t\sim N(0,0.25)$,

$\delta_t \sim N(0,0.16)$, $\varepsilon_t \sim N(0,0.16)$, $y_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} + u_t$ with standard parameter values $(\beta_1 = 1.01, \quad \beta_2 = 0.75, \quad \beta_3 = 0,25)$, were derived from the theory of production by Charles C.W. and Douglass P.H. (1928) with the following assumptions: $\beta_1 > 0, \quad 0 < \beta_2 < 1, \quad 0 < \beta_3 < 1$.

Four (4) different smoothers (i.e Epanechnikov Kernel, Gaussian Kernel, Wavelet and Polynomial Spline) are used to denoise the data in two approaches. Firstly, only the explanatory variables are denoised, later, both the dependent and explanatory variables are also denoised under three (3) different sample sizes (i.e 32, 256 and 1024). The choice of the smoothing parameter for the Kernels, Wavelet and Polynomial Spline smoothers is selected by Plug-in-method, Universal threshold and interesting range methods respectively. The regression to the denoised data is fitted and then applied to the estimators' one after the other.

Sample sizes 32, 256, and 1024 are drawn repeatedly from the model (4.1). In each case, the MSE of the estimators are computed to compare the performance of the denoised nonlinear estimators, i.e. the MSE of the denoised nonlinear least squares (DNLS) estimator, denoised nonlinear least absolute deviation (DNLAD) estimator and denoised nonlinear M- estimator are computed from 1,000 Monte Carlo samples. The analysis is carried out using R statistical package and the simulation results are summarized in the numerical tables below.

Table 1: Mean Squared Errors of the denoised nonlinear estimators when Epanechnikov Kernel is used as asmoother.

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0007606	0.0001465	0.0000866	0.0006904	0.0000968	0.0000387
	β_2	0.0000393	0.0000088	0.0000058	0.0000359	0.0000063	0.0000033
	β_3	0.0000160	0.0000020	0.0000005	0.0000169	0.0000020	0.0000005
DNLAD	β_1	0.0015072	0.0002509	0.0001993	0.0013879	0.0001309	0.0001157
	β_2	0.0000883	0.0000203	0.0000193	0.0000841	0.0000163	0.0000139
	β_3	0.0000304	0.0000027	0.0000007	0.0000307	0.0000031	0.0000009
DNM	β_1	0.0009225	0.0001690	0.0000910	0.0008680	0.0001275	0.0000455
	β_2	0.0000458	0.0000097	0.0000063	0.0000435	0.0000079	0.0000037
	β_3	0.0000203	0.0000026	0.0000006	0.0000203	0.0000026	0.0000007

Table 4: Mean Squared Errors of the denoised nonlinear estimators when Gaussian Kernel is used as smoother.

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0006835	0.0001164	0.0000670	0.0006622	0.0000775	0.0000176
	β_2	0.0000339	0.0000069	0.0000046	0.0000337	0.0000054	0.0000027
	β_3	0.0000160	0.0000020	0.0000005	0.0000160	0.0000020	0.0000005
DNLAD	β_1	0.0013939	0.0002083	0.0001719	0.0013600	0.0001309	0.0001059
	β_2	0.0000819	0.0000157	0.0000193	0.0000798	0.0000098	0.0000133
	β_3	0.0000322	0.0000026	0.0000007	0.0000305	0.0000027	0.0000009
DNM	β_1	0.0008562	0.0001407	0.0000733	0.0008331	0.0001178	0.0000393
	β_2	0.0000442	0.0000075	0.0000063	0.0000411	0.0000065	0.0000029
	β_3	0.0000203	0.0000026	0.0000006	0.0000203	0.0000026	0.0000007

Table 4.9.3: Mean Squared Errors of the denoised nonlinear estimators when Wavelet is used as smoother

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0006827	0.0000797	0.0000202	0.0006622	0.0000775	0.0000202

	β_2	0.0000353	0.0000040	0.0000011	0.0000337	0.0000040	0.0000011
	β_3	0.0000168	0.0000020	0.0000005	0.0000160	0.0000020	0.0000005
DNLAD	β_1	0.0013525	0.0001371	0.0000663	0.0013637	0.0001497	0.0000628
	β_2	0.0000730	0.0000093	0.0000085	0.0000745	0.0000104	0.0000133
	β_3	0.0000304	0.0000031	0.0000007	0.0000303	0.0000034	0.0000007
DNM	β_1	0.0008127	0.0001069	0.0000271	0.0008468	0.0001066	0.0000277
	β_2	0.0000410	0.0000070	0.0000015	0.0000423	0.0000053	0.0000029
	β_3	0.0000194	0.0000026	0.0000006	0.0000203	0.0000026	0.0000006

Table 4.9.4: Mean Squared Errors of the denoised nonlinear estimators when Polynomial Spline is used as smoother

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0006665	0.0000781	0.0000204	0.0006637	0.0000779	0.0000200
	β_2	0.0000337	0.0000041	0.0000011	0.0000325	0.0000041	0.0000010
	β_3	0.0000160	0.0000020	0.0000005	0.0000160	0.0000020	0.0000005
DNLAD	β_1	0.0013335	0.0001212	0.0000664	0.0013652	0.0001417	0.0000687
	β_2	0.0000742	0.0000102	0.0000139	0.0000773	0.0000109	0.0000079
	β_3	0.0000315	0.0000026	0.0000007	0.0000315	0.0000027	0.0000008
DNM	β_1	0.0008311	0.0001066	0.0000279	0.0008478	0.0001075	0.0000277
	β_2	0.0000411	0.0000053	0.0000015	0.0000410	0.0000059	0.0000015
	β_3	0.0000194	0.0000026	0.0000006	0.0000203	0.0000026	0.0000006

Table 1-4 show the estimated mean square error (MSE) of denoised non-linear estimators (DNLS, DNLAD and DNM) under the three different sample sizes for the two denoising approaches. Comparing the mean square error from the various smoothers used, it can be found that DNLS estimator provide smaller mean square errors under each sample size considered for the two dinoising approaches. Also it was discovered that denoising both dependent and independent variables approach had smaller mean square error under the kernel smoothers for all nonlinear estimators, while only DNLS estimator had smaller mean square error under wavelet and polynomial spline smoothers compared to denoising only independent variables approach.

CONCLUSION

This study examines the effect of denoising both dependent and independent variables on the performances of non-linear regression parameters of the estimators. The Epanecnikov, Gaussian, Wavelet and Polynomial Spline smoothers are firstly used to denoise only the independent variables and later used to denoised both dependent and independent variables under the three different sample sizes. The performances of the non-linear estimators are compared base on the mean square error criterion. The simulation study carried out for sample sizes 32, 256 and 1024 with 1000 Monte Carlo samples, showed that denoising both dependent and independent variables approach enhance the performances of the DNLS, DNLAD and DNM under Kernel smoothers but the enhancement is limited under wavelet and polynomial spline except DNLS. Therefore, denoising both dependent and independent variables approach under the three sample sizes enhanced the performances of the DNLS estimator for the four smoothers considered.

REFERENCES

1. Carroll, R. J., Ruppert, D., and Stefanski, L. A., (1995): Measurement error in nonlinear models. Chapman and hall.
2. Cheng, C. L. and Van Ness, J. W. (1999): Statistical Regression with Measurement Error. Arnold, London. pp. 3-11.
3. Cui, H.J., and Hu, T., (2010): On nonlinear regression estimator with denoised variables. Computational Statistics and Data Analysis 55, 1137-1149.
4. Fasoranbaku, O.A., and Soyombo, A.O., (2015): A comparative study of the performance of denoised non-linear regression estimators under different smoothers.
5. Fasoranbaku, O.A., and Alabi, Oluwapelumi (2016): Estimating Non-linear Regression Parameters Using Denoised Variables
6. Soyombo A. O and Fasorobaku O. A. (2015). Non-linear model: Sensitivity of its parameters on denoised regression estimators. International Journal of Advanced Scientific and Technical Research.5, 2249-9954

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