

## Instability of Ion Acoustic Wave in a Quantum Dusty Plasma Bounded in Finite Geometry

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### ABSTRACT

The instability of ion-acoustic wave in quantum dusty plasma propagating in a cylindrical waveguide filled with strongly magnetized plasma consisting of oblique streaming of holes and stationary negatively charged dust particles has been theoretically investigated using the quantum hydro-dynamical (QHD) model which includes both quantum statistical effect and the quantum diffraction effect. It is shown that ion acoustic wave become unstable in the bounded quantum dusty plasma. The growth rate of unstable wave depends on the stream velocity of holes, quantum diffraction parameter, charge imbalance parameter, inclination of stream velocity and dimension of cylindrical wave guide. Moreover, the phase velocity of the wave depends on the plasma parameters and the dimension of wave guide. Growth rate of the instability and phase velocity of wave in bounded quantum plasma are shown graphically and discussed.

**Keywords:** Instability, Ion-acoustic wave, Dusty plasma, Quantum plasma

### INTRODUCTION

Linear and nonlinear propagation of waves in various kinds plasma have been studied by any researchers for the last few decades, but a few works are done in bounded plasma system though it is important and relevant to experimental set up. One of the most important works in bounded plasma system was done by Sayal and Sharma [1] who showed that ion-Langmuir oscillations (ILO) can propagate in a plasma filled in a cylindrical wave guide even when the ions are cold and the propagating ILO in a cylindrical wave guide have dispersive characteristic and hence in the presence of nonlinearity this wave would excite solitary wave. Later, Ghosh and Das [2] investigated the effects of higher order nonlinearity and finite boundary on the propagation of electron plasma and ion acoustic K-dV solitons with the use of reductive perturbation method and a plasma wave guide geometry taking in to account of finite temperature of electrons and ions. Subsequently, Mondal et.al [3,4] and Bhattacharya et al [5,6] showed that the dimension of the cylindrical system containing the plasma does have a positive influence on the stability of the ion acoustic wave. They observed that due to the existence of the multiple mode of solution of the dispersion equation (involving Bessel function) the usual conclusion of the free space analysis can get changed completely.

In recent years there has been a great deal of interest in the investigation of various collective processes in quantum-dusty plasmas. Quantum effects may become important in a variety of environments when the plasma temperature is low and density is high. The dispersion caused by strong density correlation due to quantum fluctuations can play important role. For a quantum plasma deBroglie wavelength is comparable to the dimensions of the system. Quantum effect on the propagation of waves has been studied recently by many researchers [7-14] because of its importance in electronic devices with nano electronic components [15,16], micro plasma system [17], laser solid interaction [18]

and dense astrophysical environments [19,20].

In our present study, we have studied the instability of electro-acoustic wave in a electron-hole quantum plasma bounded in cylindrical geometry containing electron, hole and negatively charged dust particles having oblique streaming motions. We have derived the dispersion relation of the electro acoustic wave and the stability of the wave in the bounded quantum plasma has been analyzed. It is found that the electro-acoustic wave would be unstable in the bounded plasma considering the quantum effect in presence stream velocity of electrons. The variations of the instability factor and phase velocity of unstable modes with the plasma parameters has been graphically discussed for a model plasma.

### Assumptions and Basic Equations

The plasma is assumed to be immersed in an essentially infinite axial magnetic field. The magnetic field constrains the plasma particles to move only along the axis of the waveguide, which we choose as x-axis of  $(r, \theta, x)$  cylindrical coordinate system. We are interested to examine the propagation of slow modes having the phase velocities much less than the phase velocity of light and therefore the quasi-static approximation can be used.

Under the above conditions using one-dimensional QHD model the governing equations in nondimensional forms are:

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0$$

(1)

$$\mu \left( \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} \right) = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x} \left[ \frac{1}{\sqrt{n_e}} \left( \frac{\partial^2}{\partial x^2} \sqrt{n_e} \right) \right]$$

(2)

$$\frac{\partial u_h}{\partial t} + u_h \frac{\partial u_h}{\partial x} = -\frac{\partial \phi}{\partial x} - \sigma n_h \frac{\partial n_h}{\partial x} + \frac{\mu H^2}{2} \frac{\partial}{\partial x} \left[ \frac{1}{\sqrt{n_h}} \left( \frac{\partial^2}{\partial x^2} \sqrt{n_h} \right) \right] \quad (3)$$

$$\left[ \frac{1}{d^2} \nabla_{\perp}^2 + \frac{\partial^2}{\partial x^2} \right] \phi = \left[ \frac{n_e}{\delta} + \frac{Z_d n_{d0}}{n_{h0}} - n_h \right] \quad (4)$$

In the above Eqs.(1)-(4) the following normalizations have been used:

$$n_j = n_j / n_{j0}, u_j = u_j / C_s, \phi = e\varphi / (2K_B T_{Fe}), x = \omega_{ph} x / C_s, r = R, t = t \cdot \omega_{pe}$$

$$\text{where, } \omega_{pj} = (n_{j0} e^2 / \epsilon_0 m_j)^{1/2}, C_s = (2K_B T_{Fe} / m_h)^{1/2},$$

$T_{FH}$  is the electron Fermi temperature,  $n_j$ ,  $u_j$ ,  $m_j$  and  $q_j$  are respectively the perturbed number density,  $x$ -component of velocity, mass and charge of the  $j$ -th species ( $j = e$  for electrons and  $h$  for holes),  $q_e = -e$ ,  $q_h = e$ ,

$\nabla_{\perp}^2$  is the transverse Laplacian in cylindrical co-ordinates,  $\varphi$  is electrostatic potential and  $\hbar$  is the Planck's constant

divided by  $2\pi$ .  $\mu = m_e / m_h$  is the electron-hole mass ratio; in semiconductor  $\mu$  is taken as the ratio of effective mass of electrons and holes.  $\mu$  is different from for different semiconductor due to parabolicity of conduction band. But in

many cases  $\mu$  is taken to be 1.  $\delta = n_{h0} / n_{e0}$  is the charge imbalance parameter originating from dust particles,  $\delta > 1$  for negatively charged dust grains in background and  $\delta < 1$  for positively charged dust grains  $H = \hbar \omega_{pe} \delta^{1/2} / (2K_B T_{Fe})$

is a dimensionless parameter proportional to quantum diffraction,  $d = R / \sqrt{2} \lambda_e$  in which

$\lambda_e = (\epsilon_0 K_B T_{Fe} / e^2 n_{e0})^{1/2}$  is the quantum electron Debye-length. At equilibrium the charge neutrality condition is  $n_{e0} + Z_d n_{d0} = n_{h0}$ , where,  $n_{e0}$ ,  $n_{h0}$  and

$n_{d0}$  are respectively the equilibrium number density of electron, hole, and dust particles,  $Z_d$  is the number of electrons residing on the dust grain surface. The boundary condition is  $\varphi = 0$  at  $r = 1$ .

## Dispersion Relation

For the derivation of dispersion relation we assume the following perturbation

expansions of the field quantities

$$n_j = 1 + \epsilon n_j^{(1)} + \epsilon^2 n_j^{(2)} + \dots$$

$$u_j = u_j^{(0)} \cos \theta_j + \epsilon u_j^{(1)} + \epsilon^2 u_j^{(2)} + \dots$$

$$(5) \phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots$$

Further, we assume the space and time dependence of the perturbed quantities to be of the form  $\exp i(kx - \omega t)$ ,  $k$  is the wave number and  $\omega$  is wave frequency, and  $\alpha$ -dependence of the field quantities to be of the form  $\exp(i z \alpha)$ ,  $z$  being the azimuthal wave number. The streams of the electrons and holes are at an angle  $\theta_j$  with direction of propagation of the wave, i.e.,  $\vec{u}_j^{(0)} = (u_j^{(0)} \cos \theta_j, 0, u_j^{(0)} \sin \theta_j)$ .

Substituting (5) in Eqs.(1)-(4) we get the following linear dispersion relation for the ion acoustic wave

$$p_{sn}^2 = (kd)^2 \left[ -\frac{1/\delta}{k^2(1+k^2 H^2/4)} + \frac{1}{\omega_2^2 - k^2 \sigma} - 1 \right]$$

(6)

$$\text{where, } \omega_2 = \omega + k u_h^{(0)} \cos \theta_h$$

After a few algebraic steps, the dispersion relation (8) can be written in the form:

$$A_6 k^6 + A_5 k^5 + A_4 k^4 + A_3 k^3 + A_2 k^2 + A_1 k + A_0 = 0 \quad (7)$$

Where,

$$A_6 = \frac{H^2 d^2}{4} [\delta \sigma - \delta u_{oh}^2 \cos^2 \theta_h]$$

$$A_5 = \frac{H^2 d^2 \omega}{2} [\delta u_{oh} \cos \theta_h] + \frac{P_{01} H^2 d}{2} [\delta \sigma - \delta u_{oh}^2 \cos^2 \theta_h]$$

$$A_4 = H^2 d \omega P_{01} \delta u_{oh} \cos \theta_h - \frac{H^2 d^2}{4} [\delta \omega^2 + \delta]$$

$$- \frac{P_{01}^2 H^2 \delta}{4} [\sigma - \delta u_{oh}^2 \cos^2 \theta_h] + d \delta [d \sigma + u_{oh}^2 \cos^2 \theta_h]$$

$$A_3 = -\frac{P_{01}H^2d\omega^2\delta}{2} - 2dP_{01}[\delta\sigma - \delta u_{oh}^2 \cos^2 \theta_h] - 2\omega d^2[\delta u_{oh} \cos \theta_h] - \frac{H^2\omega}{2}[P_{01}^2\delta u_{oh} \cos \theta_h]$$

$$A_2 = -4d\omega P_{01}\delta u_{oh} \cos \theta_h + d^2\omega^2\delta + \frac{P_{01}^2H^2\omega^2\delta}{4} - P_{01}^2[\delta\sigma + \delta u_{oh}^2 \cos^2 \theta_h] - d^2[\delta + \sigma - u_{oh}^2 \cos^2 \theta_h]$$

$$A_1 = 2\omega^2dP_{01}\delta + 2\omega P_{01}^2\delta u_{oh} \cos \theta_h$$

$$A_0 = \omega^2P_{01}^2\delta + d^2\omega^2$$

#### 4. Instability of Wave

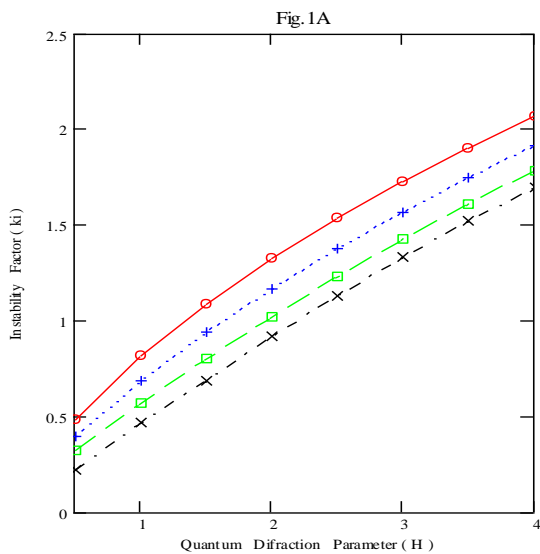


Fig.1A. Growth rate for different values of quantum diffraction parameter and

To study the instability of ion-acoustic wave in the bounded quantum plasma the dispersion relation (9) has been numerically solved using *MATCAD* software. Equation (7) is a sixth-order algebraic equation in  $k$  and correspond different modes of wave propagation. It is seen that for a particular set of values of the parameter  $\delta$ ,  $\omega$ ,  $H$ ,  $d$ ,  $u_{oh}$  and in plasma two roots are complex and four roots are real. The real roots correspond to the stable modes and imaginary parts of the complex roots correspond to the instability of ion acoustic waves when it propagates through the plasma medium. The growth rate is determined by  $|k_i|$  where  $k_i$  is imaginary part of  $k$ . In Fig.1A we have shown the variation of instability growth rate  $|k_i|$  (for one typical mode) for different values of quantum diffraction parameter  $H$  and obliqueness of stream velocity of holes in a bounded quantum dusty plasma

where The plasma parameters are  $\omega = 0.1, \sigma = 0.01, \delta = 0.1, u_0 = 0.25, p_{01} = 2.4048$ . We find that the instability growth rate increases with increase in quantum diffraction parameter but decrease with the increase of obliqueness of stream velocity.

Inclination of stream velocity. The red, blue, green and black lines represent for  $\theta_h = 0, \pi/6, \pi/4, \pi/3$ .

In Fig.1B, the phase velocity of the ion acoustic wave is plotted which shows that the phase velocity decrease with the increase of  $H$  and it increase with the inclination of the stream velocity.

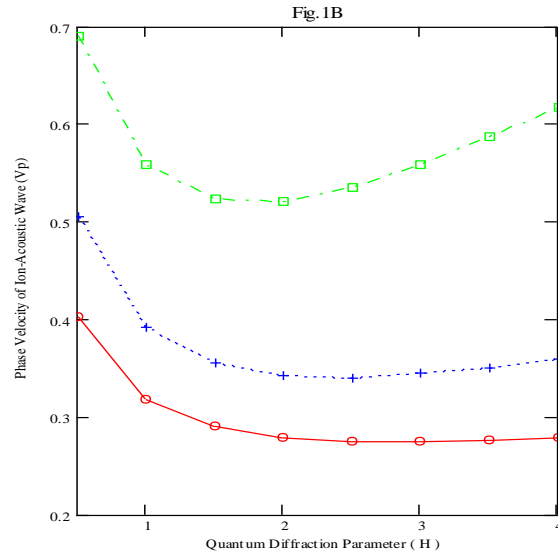


Fig. 1B Phase velocity of ion acoustic wave for different  $H$  and inclination of

stream velocity. The red, blue and green lines represent for  $\theta_h = 0, \pi/6$  and  $\pi/3$ .

Fig. 2A shows the dependence of instability growth rate for different transverse dimension of wave guide and inclination of stream velocity in a bounded quantum plasma having  $H=2.5, \sigma = 0.1, \delta = 0.5, \omega = 0.01, p_{01} = 2.4048, u_0 = 0.25$ . It is seen that the instability growth rate is lower for higher values of  $d$  and inclination  $\theta_i$ .

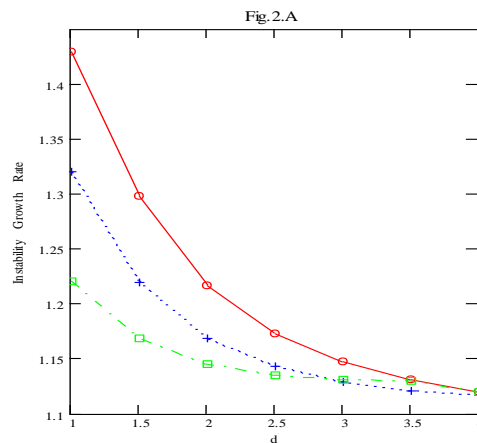


Fig.2A. Variation of instability growth rate for different transverse dimensions

of wave guide (d) and inclination ( $\theta_i$ ) of stream velocity. The red, blue and

green lines represent for  $\theta_h = 0, \pi/6, \pi/3$   
 Variation of phase velocity of the ion acoustic wave with transverse dimension of wave guide (d) has been shown in Fig.2B . It is seen the phase velocity increases with the increase of d and also with the increase of inclination of the stream velocity.

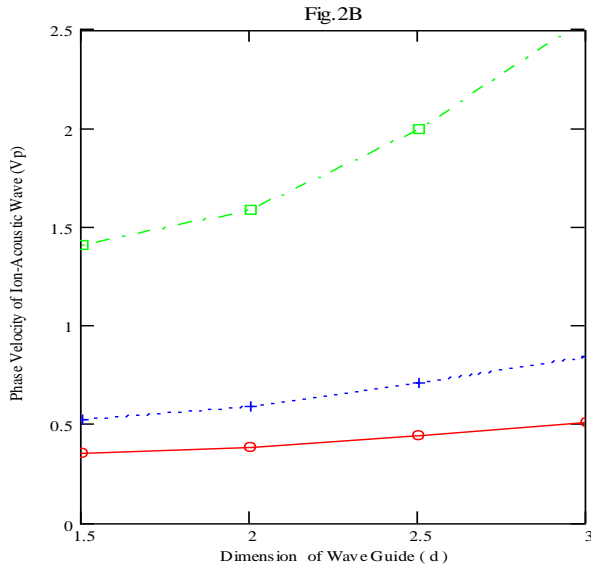


Fig. 2B . Phase velocity of ion acoustic wave for different transverse dimension of wave guide and inclination of stream velocity. The red, blue and green lines represent for  $\theta_h = 0, \pi/6,$  and  $\pi/3$ .

In Fig. 3A we have shown the dependence of instability growth rate for charge imbalance parameter  $\delta$  and inclination  $\theta$  of stream velocity in the bounded plasma where  $H=2.5, \square = 0.1, d=5, \square = 0.01, p_{01}=2.4048, u_0=0.25$ . It is seen from Fig.3A that the instability growth rate decreases with increase of dimension of wave guide (d) and increase of inclination  $\theta$  of stream velocity.

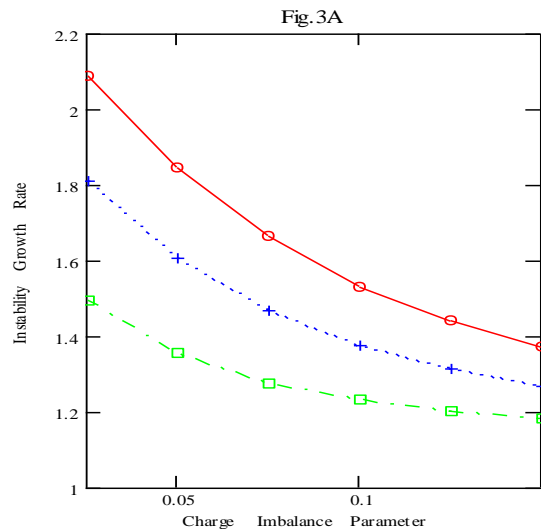


Fig. 3A. Instability growth rate of ion acoustic wave in quantum dusty plasma

for different values of H and charge imbalance parameter  $\delta$ . The red, blue and

green lines represent  $\theta_h = 0, \pi/6$  and  $\pi/3$   
 The phase velocity of ion acoustic wave shown in Fig.3B indicates that phase velocity increases with the increase of  $\delta$  and also with increase of inclination of stream velocity.

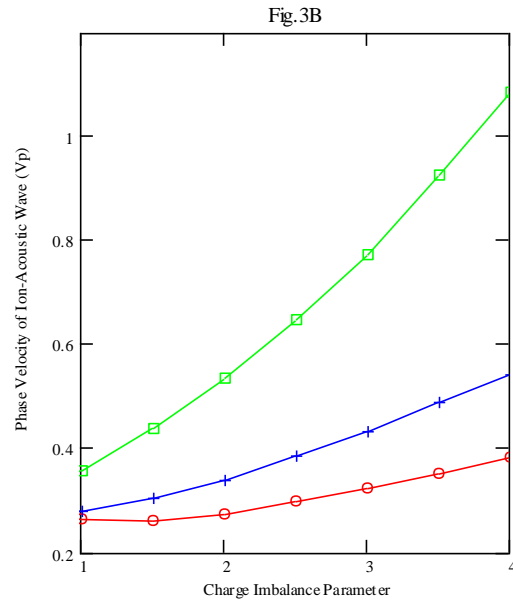


Fig. 3B-Variation of phase velocity with charge imbalance parameter

and inclination of stream velocity, The red, blue and green lines represent  $\theta_h = 0, \pi/6$  and  $\pi/4$ ,

### SUMMARY AND CONCLUDING REMARKS

In the present paper linear instability of ion-acoustic wave has been theoretically studied in an electron-hole quantum dusty plasma bounded in a cylindrical geometry considering the presence of oblique streaming motion of holes. Inclusion of streaming motion of holes plays important roles to have a number of wave modes of ion acoustic waves some of which are linearly unstable. The growth rate of this instability is shown to depend significantly on different plasma parameters such as stream velocity of plasma particles, quantum diffraction parameter, the charge imbalance parameter and inclination of stream velocity with the direction of propagating wave and also on the transverse dimension of cylindrical wave guide. Moreover, the phase velocity of the wave depends on the plasma parameters and the dimension of wave guide. The results presented in this paper might be helpful in understanding the importance of stream velocity of holes in semiconductors.

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