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Effect of Dust Streaming on Linear and Nonlinear Electrostatic Waves in Magnetized Plasma

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ABSTRACT

In this paper we have studied both linear and the nonlinear propagation of electrostatic waves in magnetized homogeneous collisionless dust-ion plasma containing Boltzmann distributed ions and highly charged streaming dust grains. From the numerical study of the linear dispersion relation it is shown that for a given wave number the wave frequency increases with increase in the streaming velocity of dust particles but the wave frequency decreases with the increase in obliqueness of the wave propagation. To study the nonlinear behavior of the wave we have derived an energy integral equation by using Sagdeev pseudo potential technique. Effects of dust streaming and obliqueness of propagation on the condition of formation and profile of possible solitary wave structure have also been numerically analyzed and discussed.

Keywords: Electrostatic wave, Dust streaming, Sagdeev potential, Solitary wave.

INTRODUCTION

In recent years the study of dust acoustic solitary waves (DASWs) has become an important topic of plasma research. This is due to its relevance to some space and laboratory plasmas [1-10] such as cometary tails, planetary rings, lower parts of the ionosphere, interstellar clouds, auroras, glow discharges, RF plasma etching, wall region of tokamak plasma, laboratory devices for the production of many modern materials, etc. Dusty plasma can also be found in the Earth's ionosphere. An exciting area of recent research is the study of electrostatic and electromagnetic waves in dusty plasmas. Existence of one of these waves, called dust acoustic wave (DAW) was theoretically predicted by Rao et al [11]. This wave is a very low-frequency acoustic-like wave. The experimental verification of the existence of this kind of waves was done by Chu et al [12]. Moreover, theoretical prediction of the existence of dust ion-acoustic waves was done by Shukla and his collaborators [13]. A number of experimental [14, 15] and theoretical [16, 17] studies have also been made on the linear and nonlinear properties of DAWs in unmagnetized dusty plasmas. But most of the dusty plasmas in the laboratory, space and astrophysical environments are subjected to an external magnetic field. So naturally it is interesting to investigate various aspects of linear and nonlinear dust-acoustic waves in a magnetized plasma [18, 19]. Most of the dusty plasmas are of dust-ion-electron type. But there are some situations where dust-ion plasma model can be used. This model has relevance to Saturn's F-ring [20], surroundings of Halley's comet [21] and some laboratory plasma environments [22].

Recently nonlinear propagation of the dust-acoustic waves has been studied in unmagnetized [23] as well as magnetized [24-26] dust-ion plasma. Kotsarenko [24] and Mamun [25] used reductive perturbation technique which is valid for small but finite amplitude waves. Farid et al [26] considered magnetized dust-ion plasma and studied nonlinear propagation of the coupled dust-acoustic and dust-cyclotron waves by means of the Sagdeev potential approach which is applicable to arbitrary amplitude waves. They have found the existence of solitary waves with a negative potential. At the same time, they have shown that some properties of the solitary waves viz. amplitude, width etc are significantly modified by the external magnetic field and the obliqueness of wave propagation. But they did not take into account the effect of dust-streaming on the solitary wave propagation.

There are many situations in space and astrophysical plasmas where dust streaming can occur [27]. It has been found that dust streaming has considerable effect on the formation of dust-acoustic solitary wave structures. A number of works have been reported relating to the effect of dust streaming in unmagnetized plasma. In a recent paper Mahmood and Saleem [28] have studied the effects of dust streaming on the nonlinear dust acoustic wave in homogeneous dust-ion and dust-ion-electron plasmas without considering the effect of external magnetic field.

The purpose of the present paper is to study the linear and nonlinear propagation of dust acoustic wave in a dust-ion plasma including both the effects of dust streaming and the external magnetic field. It has been shown that dust streaming, obliqueness of propagation and the strength of magnetic field have considerable effect on the linear properties, condition of formation and profile of possible DAW solitary wave structure.

The manuscript is organized as follows: In section II, we have presented the basic equations governing the dynamics of the waves. In section III, we have made linear as well as nonlinear analysis of the plasma waves. In section IV we have presented and discussed numerical results.

MATHEMATICAL FORMULATION

We consider a two-component homogeneous magnetized dust-ion plasma containing singly charged Boltzmann distributed ions and negatively charged, massive, micron-size dust grains. In this two-component dust-ion plasma model we assume that most of the electrons from the ambient plasma are attached to the surface of the dust grains so that we have $n_{e0} << z_d n_{d0}$; n_{eo}, n_{do} being the unperturbed electron and dust particle number density respectively and z$_d$ is the number of electrons resting on the surface of a dust particle. In the state of equilibrium, we have $n_{i0} \approx z_d\ n_{d0}$. We further assume that the plasma is immersed in a uniform external magnetic field $\vec{B}_0 = \hat{e}_z B_0$, where $\hat{e}_z$ is the unit vector along the z-axis. We also assume that the grain size is much smaller than dust cyclotron radius. We concentrate on one dimensional wave propagation and take the dust particles to be cold and streaming with some constant velocity $\vec{v}_{d0} = \hat{e}_z v_{d0}$ along the magnetic field. The basic equations that govern the dynamics of DAW in this two-component magnetized dust-ion plasma are:

$$\frac{\partial n_d}{\partial t} + \vec{\nabla}.(n_d \vec{v}_d) = 0 \tag{1}$$

$$\frac{\partial \vec{v}_d}{\partial t} + (\vec{v}_d.\vec{\nabla})\vec{v}_d = \left(\frac{ez_d}{m_d}\right)\vec{\nabla}\phi - \omega_{cd}(\vec{v}_d \times \hat{e}) \tag{2}$$

$$n_i = n_{i0}\left(\frac{-e\phi}{k_B T_i}\right) \tag{3}$$

Here n$_d$ and v$_d$ are the dust number density and the dust fluid velocity respectively, $\omega_{cd}\left(=\frac{ez_d B_0}{m_d c}\right)$ is the dust-cyclotron frequency, c is the velocity of light in free space, e is the magnitude of the electronic charge, n_i is the ion number density, T_i is the ion temperature, k_B is the Boltzmann constant and ϕ is the electrostatic potential. The equations (1)-(3) are supplemented by the quasi-neutrality condition

$$n_i = z_d n_d \tag{4}$$

LINEAR INSTABILITY

To derive the dispersion law we expand the plasma variables n_d, v_d and ϕ around their equilibrium values as

$$n_d = n_{d0} + n_{d1};\ \vec{v}_d = \vec{v}_{d0} + \vec{v}_{d1} \text{ and } \phi = \phi_1 \tag{5}$$

Linearzing Eqs.(1)- (5) and assuming space-time dependence of the first order perturbations to be proportional to *exp*[i(kr-ωt)] where ω and k are the normalized wave frequency and wave number respectively and we consider that disturbance is confined in x-z plane. We have normalized wave frequency (ω) by ω$_{cd}$, wave number (k) by ω$_{cd}$/c$_d$ and dust fluid velocity (v_d) by c_d. We obtain the following linear dispersion law:

$$k^2(\omega - k_z v_{dz0})^2 - k_z^2 + (\omega - k_z v_{dz0})^2\left[1 - (\omega - k_z v_{dz0})^2\right] = 0 \tag{6}$$

Here, $c_d = \sqrt{\frac{K_B T_i z_d}{m_d}}$ is the dust acoustic speed and k$_z$=k·cosθ , where θ being the angle between $\hat{k}$ and $\vec{B}_0$. Now it is interesting to note that the dispersion relation (6), in absence of dust streaming i.e., for $v_{dz0} = 0$, reduces to the dispersion law obtained in reference [26]. The dispersion law given by (6) can also be expressed as

$$a\omega^4 + b\omega^3 + c\omega^2 + d\omega + e = 0 \tag{7}$$

here, $a = -1$; $b = 4k_z v_{dz0}$; $c = 1 + k^2 - 6k_z^2 v_{dz0}^2$;

$d = 2k_z v_{dz0}\left[2k_z^2 v_{dz0}^2 - k^2 - 1\right]$; and

$e = k_z^2\left[v_{dz0}^2\left(1 + k^2 - k_z^2 v_{dz0}^2\right) - 1\right]$

Solutions of equation (7) will give various modes of wave propagation in the plasma. Thus we get two obliquely propagating modes called the dust acoustic mode and the dust cyclotron mode. In presence of dust streaming the dispersion law gets modified and in fact it has significant effect on the linear and nonlinear propagation of the wave. Equation (7) is an algebraic equation of fourth order in wave frequency. Its solutions will give various modes of wave propagation in the plasma.

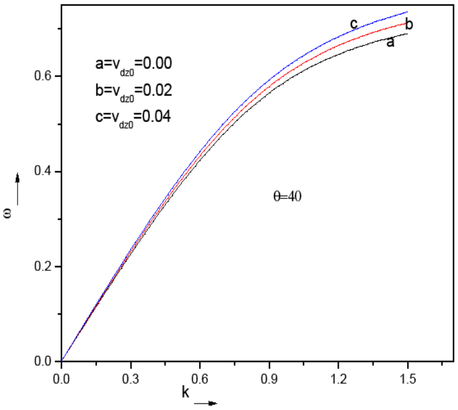

Fig.1- Variation of wave frequency (ω) with wave number k for different values of v$_{dz0}$

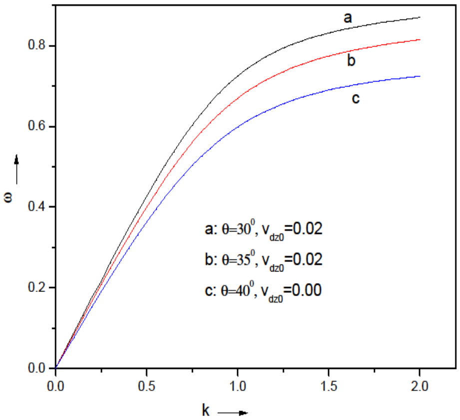

Fig.2- Variation of wave frequency (ω) with wave-number k for different values of obliqueness (k$_z$)

NONLINEAR PROPAGATION:

With a view to study the obliquely propagating nonlinear waves in our two-component dust-ion magnetized plasma. We introduce the following normalized variables:

$$N_d = \frac{n_d}{n_{d0}}; \vec{u}_d = \frac{\vec{v}_d}{c_d}; \psi = \frac{e\phi}{K_B T_i}; \vec{R} = \frac{\vec{r}}{\rho_d} \text{ and}$$

$$\tau = \omega_{cd} t \qquad (8)$$

$$\text{Here, } \rho = \frac{c_d}{\omega_{cd}}; \vec{r} = \hat{i}x + \hat{k}z; \vec{R} = \hat{i}X + \hat{k}Z$$

Now we look for the solutions of Eqs. (1)- (3) which are dependent on *X*, *Z* and τthrough a single variable ηdefined by

$$\eta = \frac{1}{M}(\alpha X + \gamma Z - \tau M) \qquad (9)$$

Here *M*=vp/cd, $\alpha=k_x/k$, $\gamma=k_z/k$ and $k = \sqrt{k_x^2 + k_z^2}$ with the use of (9) and the boundary conditions $N_d \to 1$, $u_{dz} \to u_{dz0}$, $u_{dx} \to 0$ as $\eta \to \pm\infty$, equation (1) yields

$$(-M + \alpha u_{dx} + \gamma u_{dz}) N_d = \gamma u_{dz0} - M \qquad (10)$$

Equations (2) and (3), with the use of (9) get transformed into

$$(-M + \alpha u_{dx} + \gamma u_{dz}) \frac{du_{dx}}{d\eta} = \alpha \frac{d\psi}{d\eta} - M u_{dy} \qquad (11)$$

$$(-M + \alpha u_{dx} + \gamma u_{dz}) \frac{du_{dy}}{d\eta} = M u_{dx} \qquad (12)$$

$$(-M + \alpha u_{dx} + \gamma u_{dz}) \frac{du_{dz}}{d\eta} = \gamma \frac{d\psi}{d\eta} \qquad (13)$$

and $$N_d = e^{-\psi} \qquad (14)$$

Combining (10) and (14) and applying the boundary conditions $Nd \to 1, \psi \to 0$ as $\eta \to \pm\infty$ one gets

$$u_{dz} = u_{dz0} + \frac{\gamma}{\Gamma}\left(1 - e^{-\psi}\right) \qquad (15)$$

Here, $\Gamma = \gamma u_{dz0} - M$

Eliminating u_{dx}, u_{dy} and u_{dz} from Eqs. (11)- (13), using (15) and applying the boundary conditions $\frac{d\psi}{d\eta} \to 0$, $\psi \to 0$ as $\eta \to \pm\infty$ we get the "energy integral" equation of the form

$$\frac{1}{2}\left(\frac{d\psi}{d\eta}\right)^2 + V(\psi) = 0 \qquad (16)$$

Here,

$$V(\psi) = \frac{M^2}{\Gamma^2(1-\Gamma^2 e^{2\psi})^2} \begin{bmatrix} \Gamma^2\left(1-\psi-e^{-\psi}\right) + \frac{\gamma^2}{2}\left(e^{-2\psi} - 2e^{-\psi} + 1\right) \\ + \frac{\Gamma^4}{2}\left(e^{2\psi} - 2e^{\psi} + 1\right) - \gamma^2\Gamma^2\left(e^{\psi} - \psi - 1\right) \end{bmatrix} \qquad (17)$$

$V(\psi)$ is called the Sagdeev potential [20]. The conditions for the stationary localized solution of equation (16) require that $V(0) = V(\psi_0) = \left(\frac{\partial V}{\partial \psi}\right)_{\psi=0} = 0$ (where ψ_0 can have value>1or <1 and it is a point where the curve crosses the ψ-axis). The solitary wave solution of Eq. (16) exists only if (i) $\left(\frac{\partial^2 V}{\partial \psi^2}\right)_{\psi=0} \langle 0$, so that the fixed point at origin is unstable and (ii) $\left(\frac{\partial^3 V}{\partial \psi^3}\right)_{\psi=0} \rangle 0$ or (<0) for solitary wave with $\psi > 0$ or < 0.

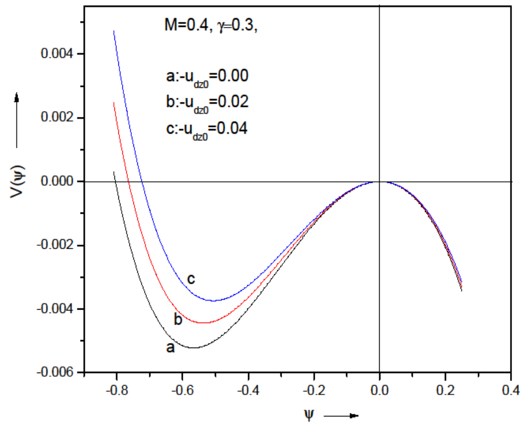

Fig. 3- Sagdeev potential profile for different values of dust Streaming velocity u_{dz0}

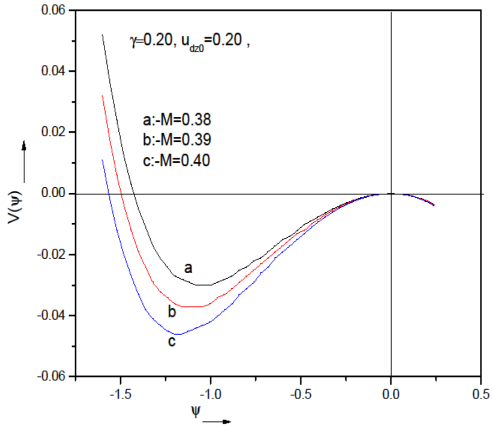

Fig. 4- Sagdeev potential profile for different values of M

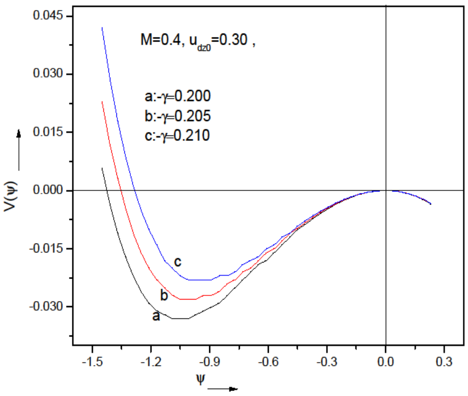

Fig.5- Sagdeev potential profile for different values of γ

RESULTS AND DISCUSSION

\In this paper we have numerically studied the effect of dust streaming on dust acoustic wave in a magnetized dust-ion plasma. The linear dispersion equation is of the fourth order in wave frequency. All the roots of this equation have been found to be real for different values of dust streaming velocity (v_{dzo}) and wave number (k). Out of these roots, two roots are negative which signifies the backward going wave and remaining two positive roots represent dust acoustic mode and the dust cyclotron mode. We have considered the dust acoustic mode. In Fig.1 we show the effect of dust streaming on the linear dispersion character of the dust acoustic mode. It shows that for a given wave number the wave frequency increases with increase in dust streaming velocity. Figure 2 shows the effect of obliqueness of wave propagation on its linear dispersion character. It shows that for a given wave number the wave frequency decreases with increase in the angle of propagation with respect to the external magnetic field. Using Sagdeev potential approach we have predicted the formation solitary wave structure and studied the effect of dust streaming and other parameters on this solitary wave structure. In Fig. 3 we show the Sagdeev potential profile for different values of dust streaming velocity keeping the values of M and γ fixed. It shows that the amplitude of solitary structure and depth of Sagdeev potential both decreases with increase in dust streaming velocity .Thus dust steaming plays a destructive role in the formation of solitary structure in magnetized dust-ion plasma. Note that the Sagdeev potential structure is formed on the negative side of ψ -axis. This is due to the fact that the dust particles are assumed to be negatively charged. In presence of an ambient magnetic field the solitary wave structures are generated when $M^2<1$[26] though the condition to be satisfied for the formation of the DAW as well as the IAW solitary structures is, in general, $M^2>1$. In Fig. 4 we show the Sagdeev potential profile for different values of Mach number M keeping the values of dust streaming velocity and γ fixed. It shows that the amplitude of solitary structure and depth of Sadie potential both increase with increase in Mach number. In Fig. 5 we show the dependence of the Sagdeev potential profile on obliqueness of wave propagation with respect to external magnetic field. Figure 5 shows that for fixed values of Mach number M and dust steaming velocity u_{dz0} along the magnetic field, the amplitude of the solitary wave increases with increase in obliqueness of propagation (i. e. decrease in the value of γ). For larger values of γ solitary waves are less likely to be excited in the simple dust-ion plasma that we have considered.

Finally we would like to point out that though we have considered a simple plasma model the results will be useful to understand more complex systems in space plasmas; particularly this model has relevance to Saturn's F-ring, surroundings of Halley's comet and some laboratory plasma environments.

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